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Hedging for Profit: A Novel Approach to Diversification

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The key to mitigating equity risk in portfolios is investing in assets that are negatively correlated with equity markets yet exhibit the potential for a positive expected return. For most of the past 20-plus years, bonds have fulfilled this role in portfolios, aided by a substantial tailwind of stable or falling inflation. On a forward-looking basis, bonds will continue to be a key part of portfolios, but the potential for both positive expected return and negative correlation with equities may be tested at times. Beyond fixed income, the search for positive-expected-return, risk-mitigating assets becomes more challenging.

In this piece, we first review the pros and cons of some other approaches in current use. We then derive some noteworthy portfolio effects achieved by combining investments with negative correlation and positive expected return potential. Similar to optimal portfolio theory, in which diversification is the only "free lunch," diversifying the approach to equity risk mitigation may be more efficient than using any single method.

POPULAR APPROACHES TO RISK MITIGATION

Investors have come to accept three general types of equity-risk-mitigation strategies: a) fixed income, b) trend-following strategies and c) tail-risk hedging via explicit option buying. The key trade-off among them is the "cost" of the risk-mitigation approach versus the level of assurance that the strategy will perform as expected in a declining-equity-market scenario. As noted above, fixed income may well continue to fulfill the role of an asset with a positive expected return and a negative correlation, but because both those properties may at times be tested, investors should not rely solely on fixed income. Tail-risk hedging all but guarantees the hedging properties (up to counterparty risk) but typically has a negative expected return

due to the cost of maintaining the options contracts. Trend-following strategies are the third popular choice. They have demonstrated attractive performance during historical equity market drawdowns due to their ability to turn rapidly and decisively (see Figure 2), and to short markets that are going down, while still delivering a positive average return across long periods.

The experience of trend-following as an alternative strategy with the desired characteristics of positive expected return and negative expected correlation leads us to consider other alternative risk-premia-style strategies. Before that, however, we will delve into more detail on the pros and cons of each of these three equity-risk-mitigation strategies.

FIXED INCOME

The use of fixed income as an equity-risk-mitigation strategy is perhaps the most prominent example of a strategy that delivers the potential for positive returns and shows negative correlation with equities. This success isn't without risk, as correlations with equities vary over time and can change abruptly, often without warning. Furthermore, with the secular decline in yields since the early 1980s, a large investor base fears a reversion to high yields as unemployment declines and money supply expands.

The orange line in Figure 1 shows a one-year rolling estimate of the correlation of 10-year U.S. Treasury returns and S&P 500 returns, computed weekly (right-hand axis). From approximately 1970 to 1995, the correlation ranged between zero and 0.6 – a strongly positive relationship that implies that the addition of fixed income to an equity-centric portfolio would not have been risk mitigating but instead risk additive. In the late 1990s, this correlation underwent a dramatic change and shifted to being strongly negative with bouts of positive correlation. This highlights an important risk underlying the use of fixed income as a hedge to equities: Correlations can shift abruptly[1] and sometimes permanently (Johnson, Naik, Pedersen, Sapra 2013). To take two simple examples, this correlation shift could happen on the back of a permanent inflation shock or forced liquidation of risk parity funds.

Separately, the blue line in Figure 1 shows the estimated 10-year U.S. Treasury bond term premium[2] from the New York Federal Reserve, which is a measure of expected excess bond returns

(left-hand axis). Since the early 1980s, the term premium has generally compressed toward zero, which has coincided with strong bond returns. Current estimates for the term premium on 10-year Treasury bonds are close to zero, highlighting that fixed income has become less attractive as a solution for mitigating equity risk in portfolios (Adrian, Crump, Moench 2013).

Figure 1: The evolution of term premium estimates in 10-year U.S. Treasury bonds, and the correlation between 10-year Treasury yield changes and the S&P 500

Term premium and equity correlation
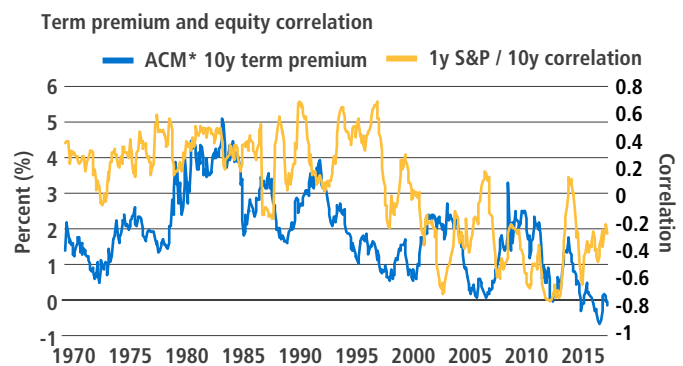
*ACM refers to Treasury term premia estimates by Federal Reserve Bank of New York economists Tobias Adrian, Richard Crump and Emanuel Moench.
Source: PIMCO and Bloomberg as of 30 April 2017

TREND-FOLLOWING

Trend-following strategies benefit from persistent trends in prices across major markets and have a return profile that is long volatility, akin to buying a straddle (Fung and Hsieh). Because they trade in the underlying assets directly, they are, in a sense, not subject to the drag from paying option premia, so returns historically have been positive, on average. Furthermore, due to their ability to short multiple markets simultaneously, they have performed especially strongly during more extended equity market declines. Figure 2 shows annual returns of the SG Trend Index of the 10 largest trend-following funds versus the annual returns of the S&P 500 from 2000 to 2016. Over this period, the correlation of annual returns is 0.43, and the diversification effects in 2002 and 2008 are especially striking. At the same time, the average returns of the two series are similar, about 6% per year.

Figure 2: Comparison of annual returns of the S&P 500 Index and the SG Trend Index
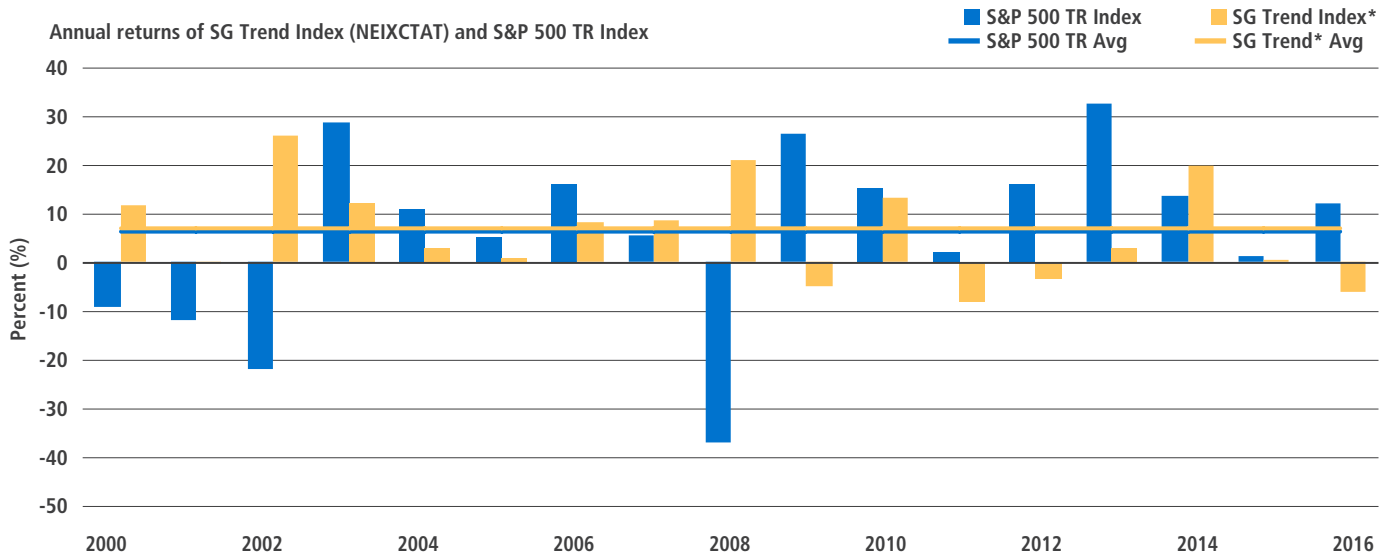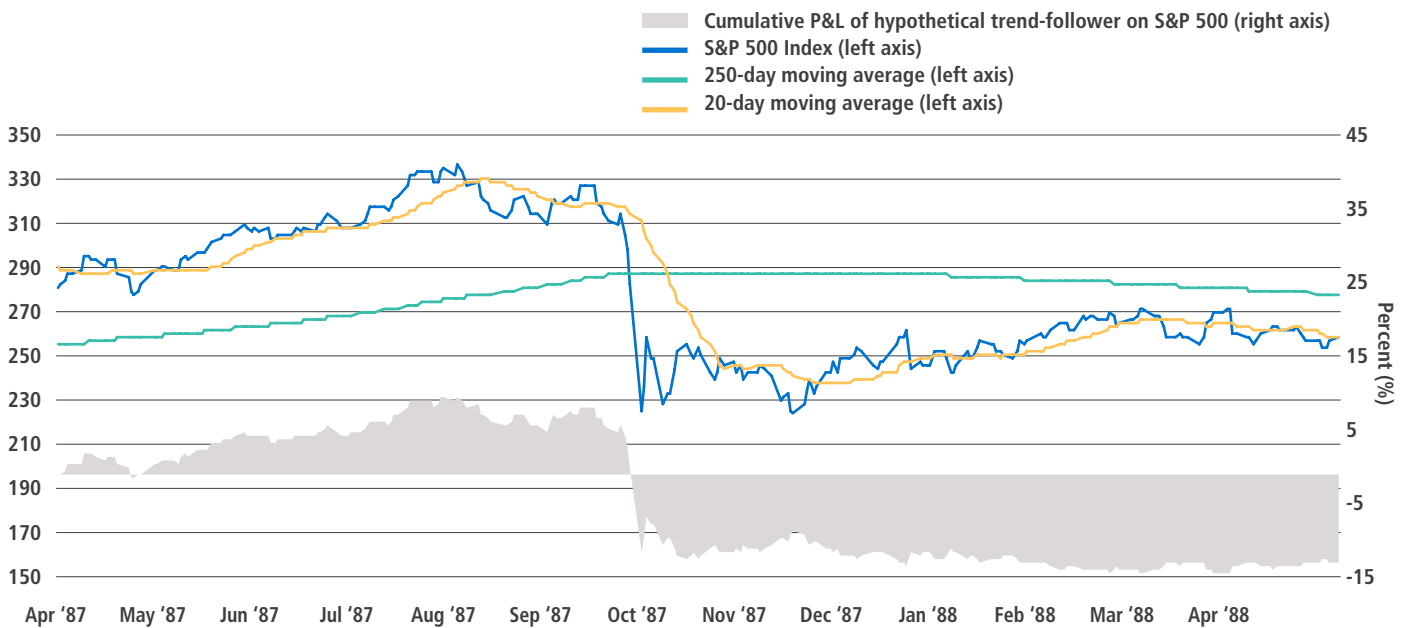
Annual returns of SG Trend Index (NEIXCTAT) and S&P 500 TR Index

Legend:
- ■ S&P 500 TR Index
- ■ SG Trend Index*
- — S&P 500 TR Avg
- — SG Trend* Avg

Source: Bloomberg and PIMCO as of 31 December 2016

Figure 3: Hypothetical returns of a simple trend-following strategy over the October 1987 market crash

Legend:
- ▨ Cumulative P&L of hypothetical trend-follower on S&P 500 (right axis)
- — S&P 500 Index (left axis)
- — 250-day moving average (left axis)
- — 20-day moving average (left axis)

Source: Bloomberg and PIMCO as of 30 June 2017. **Hypothetical example for illustrative purposes only.** Refer to disclosure for additional detail on the trend-follower model.

Although these characteristics are highly attractive for an equity-risk-mitigation strategy, using trend-following alone for this purpose has some drawbacks. Most notably, trend-followers typically require a week or two to adjust positions. For this reason, they are susceptible to gapping markets. The starkest historical example would be the October 1987 "Black Monday" crash. Though we do not have actual returns of a live trend-follower over that period, Figure 3 shows the hypothetical results of a simple trend-follower model of the S&P 500 (using a 20-day/250-day moving average crossover signal to determine longs and shorts). We see, for at least the six months prior to Black Monday, that the hypothetical trend-follower would have been long equities (since the 20-day moving average was above the 250-day moving average throughout that time) and had positive cumulative profit from April 1987 to the October crash. However, this model would have remained long the S&P 500 until the end of October 1987, when the 20-day moving average crossed back through the 250-day average to go short – too late to avoid the sharp losses in the week of the crash. Of course, actual trend-followers likely would have been better diversified across many markets, but this example highlights the gap risk these strategies can face.

TAIL-RISK HEDGING

Among the various approaches to mitigating equity risk in a portfolio, tail-risk hedging offers the highest confidence in delivering returns when needed because it is explicitly linked to the degree of decline in equities. Unlike fixed income, which relies on correlations, and trend-following, which relies on timing, tail-risk hedging relies only on defining the magnitude of the decline and the time horizon for which protection is desired. Naturally, though, this comes at a cost.

A put option is the ultimate hedge asset. To assess the expected return on a put option, we can calculate its beta. It can be shown (see Appendix 1) that the beta of a put is equal to the put elasticity times the beta of the underlying asset:

$$\beta_{put} = \frac{S}{P} \frac{\partial P}{\partial S} \beta_{underlying} \qquad (1)$$

where S and P are the values of the underlying asset and the put option, respectively, and $\frac{\partial P}{\partial S}$ is the delta of the put.

As a simple illustration, let us compute the expected return on a one-year at-the-money forward (ATMF) put on the S&P 500. Neglecting the dividend yield, a good approximation for the beta of a one-year ATMF put is $\beta_{put} \approx -\frac{1.25}{\sigma} + \frac{1}{2}$, $\qquad$ (2)

where σ is the volatility of the S&P 500. With $\sigma = 18\%$, the put delta is -0.46. Assuming the beta of the S&P 500 is 1, the beta of the put is -6.44.

With the risk-free rate r at 1% and the equity risk premium (ERP) at 3%, the expected return on the put is

$$E(r_{put}) = r + \beta_{put} * ERP = -18.34\%. \quad (3)$$

This number means that a put buyer who pays one dollar to hedge tail risks will recover, on average, 83.24 cents – that is, $e^{-0.1834}$, after one year.

The derivation reveals that expected returns to tail-risk hedging are highly dependent on valuations. An interesting implication, contrary to the perspective that tail-risk hedging is always expensive, is that put options on assets that display a negative risk premium can have significantly positive expected returns. For example, if, on the back of a pessimistic outlook, we were to expect an ERP of -5% over a given time horizon, then the expected return on the put, using similar numbers as above, would be 33.2%.

This dependence on valuations shows that investors shouldn't dismiss the role of explicit tail-risk hedging, especially at certain stages of the business cycle. Furthermore, the example makes clear that the ability to substitute hedges in correlated markets in which options are cheaper due to relative valuations can potentially provide significant return advantages.

ALTERNATIVE RISK PREMIA

There's value in expanding the search to other alternative risk premia. In some cases, these strategies can be readily adapted to tilt to a negative equity beta without giving up all the expected return.

We give two such examples from strategy indexes available on Bloomberg: 1) the low-beta equity-risk-premium strategy and 2) the roll-yield strategy in commodities. In each case, we find a natural means to adapt the strategy to tilt it toward having a negative (rather than zero) expected correlation with equities.

Table 1: Properties of selected risk-mitigation strategies, 31 December 1999 to 28 April 2017

	10y Treasury futures	SG Trend Index	S&P Low Volatility	Short commodity negative carry	Equity risk mitigation
Percent allocation	50	21	30	14	100
Sharpe	0.77	0.30	0.39	0.29	0.75
Annualized return	4.6%	4.2%	3.9%	6.3%	5.2%
Annualized volatility	6.0%	14.1%	9.9%	21.5%	7.0%
Corr w/ MSCI World	-0.27	-0.09	-0.65	-0.36	-0.59

Source: Bloomberg and PIMCO as of 28 April 2017. Based on monthly returns. **Hypothetical example for illustrative purposes only.**

The well-known low-beta equity-risk-premium strategy goes long stocks with the smallest betas to the market-cap-weighted index and short the market-cap-weighted index. For the purposes of our example, we construct a strategy that is 60% long the RAE Low Volatility US model portfolio (ticker: RALVEIUT), 40% long the RAE Low Volatility International model portfolio (ticker: RALVEIIT) and 100% short the MSCI World Index. This strategy is approximately neutral to industry- and country-specific risks yet exhibits a negative beta to the market-cap-weighted index, as the long portfolio has less market beta than the market-cap-weighted short. (The typical beta-neutral implementation of this risk-premium strategy overweights the long side to match the beta of the short side.)

The second strategy we consider takes short positions in commodities that tend to exhibit a negative average roll yield (e.g., negative carry). We construct an index that is equally weighted to a collection of short commodities[3] that includes natural gas, soybean oil, wheat, corn, cotton and coffee. Because commodities tend to be positively correlated with equities, by going short these commodities, the strategy will have a negative correlation. Furthermore, the roll yield, which is the relative price of near dated commodity contracts versus long-expiry contracts, is a well-known alternative risk premium (Gorton, Hayashi, Rouwenhorst 2012).

Table 1 reports the historical properties of these two strategies from 31 December 1999 to 28 April 2017. (We include for comparison both a simple fixed income proxy (the U.S. 10-year Treasury Futures Excess Return Index) and the trend-following (the SG Trend Index of the largest trend-following firms)).

To highlight the power of combining these types of strategies – which leads us to the interesting portfolio results derived in the next section – we include a simple combination of these four strategies, constructed by taking the average returns of the four, with each scaled to 3% stand-alone annual volatility.

Table 1 shows that each equity-risk-mitigation strategy satisfies the equity correlation and positive return criteria. Combining the strategies so that each strategy is scaled to an equal 3% volatility, the correlation of the portfolio of strategies is -0.6. As is well known when combining strategies with low correlations, the Sharpe ratio of the portfolio of strategies (in this case, 0.82) is well above the average of the Sharpe ratios of the individual strategies (in this case, the average is about 0.4). Furthermore, the portfolio correlation with equities is substantially more negative than the -0.35 average of the individual strategies' correlations with equities.

This last result is striking and powerful. In the next section, we show that this is, in fact, a general portfolio result under certain conditions. Put simply, combining risk-mitigation strategies benefits from diversification in returns *and* in correlations.

Figure 4: Rolling 2-year correlation of portfolio with MSCI World, and average correlation of individual strategies with MSCI World, 31 December 2001 to 31 December 2016
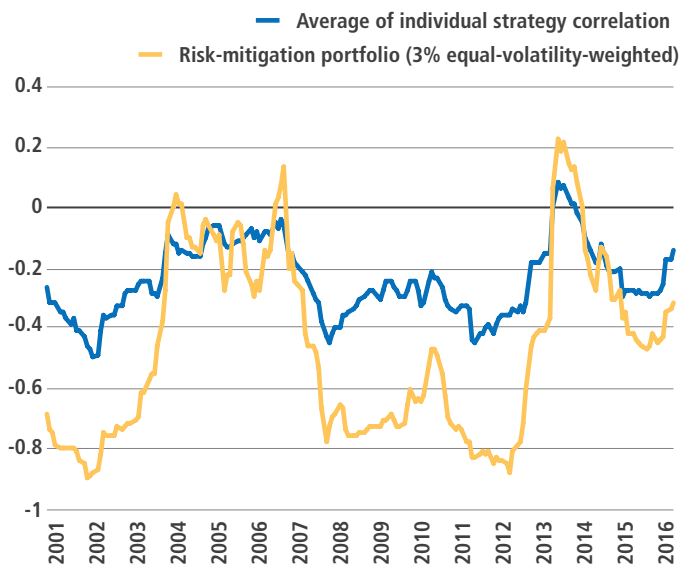
Source: PIMCO as of 31 December 2016. **Hypothetical example for illustrative purposes only.**

Figure 4 provides additional perspective on the magnification properties of the equal 3% volatility weighted risk mitigation portfolio ("risk-mitigation portfolio") correlation relative to the average of the individual strategy correlations. Over time, the correlation is substantially more negative than the average, implying that there is more certainty that a portfolio of risk-mitigation strategies will perform during equity market declines.

Bringing these theoretical and empirical results together, we believe that a portfolio of risk-mitigation strategies such as these would be more robust to time-varying equity correlations, valuations and gap risk than any single approach on its own.

Before deriving the power of combining strategies with positive return and negative equity correlation, it is useful to quantify the benefits these portfolios may have when combined with equities.

Consider an investor who allocates 80% to equities and 20% to the risk-mitigation portfolio . Suppose that the expected return on equities is 5%, the volatility of equities is 15%, and the volatility of the risk-mitigation portfolio is 10%. Table 4A in Appendix 4 shows the equivalent Sharpe ratio required on a 20% allocation to an alternative investment portfolio that has a volatility of 10% and that is uncorrelated with equities, in order to be indifferent between the risk-mitigation portfolio and the alternative investment portfolio. Here we suppose that indifference is defined by the Sharpe ratio of the overall portfolio that combines 80% in equities and 20% in nonequity investments.

For the case in which the risk-mitigation portfolio delivers a -0.5 correlation with equities and a 0.5 Sharpe ratio, an investor would prefer a portfolio invested 80% in equities and 20% in a zero-equity-correlation alternative strategy to a portfolio invested 80% in equities and 20% in the risk-mitigation portfolio only if the alternative strategy had an expected Sharpe ratio above 0.73. This is substantially above the more reasonable 0.5 Sharpe ratio of the risk-mitigation strategy.

This example highlights the power of the risk-mitigation portfolio in a broader asset allocation. By targeting both the numerator (return) and the denominator (diversification) of the Sharpe ratio, the risk-mitigation portfolio maximizes its marginal impact.

Now we derive other portfolio properties of combining these risk-mitigation strategies. Consider two strategies, denoted X and Y (e.g., trend-following and fixed income), that are both scaled to the same level of volatility. Let Z represent equities (e.g., the S&P 500). Appendix 3 shows that the correlation of a portfolio of a sum of strategies that are each scaled to contribute an equal amount of volatility is

$$\rho_{X+Y,Z} = \left(\frac{2}{\sqrt{2(1+\rho_{X,Y})}} \right) \bar{\rho}_Z \qquad (4)$$

where $\bar{\rho}_z = (\rho_{X,Z} + \rho_{Y,Z})/2$. Because $\rho_{X,Y} < 1$, the coefficient in front of $\bar{\rho}_z$ is always greater than 1. For risk-mitigation strategies where $\bar{\rho}_z < 0$, a portfolio consisting of the sum of strategies that are negatively correlated with equities is more negatively correlated than the average of the strategy correlations. Furthermore, this highlights a convexity property: The optimal approach to designing an equity-risk-mitigation portfolio is to find strategies that are uncorrelated with one another yet negatively correlated with equities.[4]

The reason this works is simple. Correlations correspond to the covariance divided by the product of the volatilities. The volatility of the portfolio of risk-mitigation strategies is less than the sum of the volatilities of the individual strategies; however, the covariance of the portfolio of strategies with equities is linear in the sum of the covariances.

As an example of the power of combining equity-risk-mitigation strategies, consider a portfolio consisting of N strategies and assume the average of the cross-correlation of these N strategies to one another is 0.1 and the average cross-correlation with equities is -0.25. These assumptions seem reasonable, as many candidate equity-risk-mitigation strategies are weakly correlated with one another and weakly correlated with equities, on average (with substantial time variation in the correlation). Furthermore, assume that each strategy has a Sharpe ratio of 0.2, so that the equity-risk-mitigation strategies satisfy the criterion of having, on average, a positive expected return. If each strategy is scaled for 3% volatility, then note how the correlation of the portfolio of strategies with equities scales with the number of strategies (see Table 2).

Table 2: Effects on portfolio equity correlation and Sharpe ratio of combining risk-mitigation strategies

Number of strategies	Equity correlation	Sharpe ratio
1	-0.25	0.20
2	-0.34	0.27
3	-0.40	0.32
4	-0.44	0.35
5	-0.47	0.38
6	-0.50	0.40

Source: PIMCO. **Hypothetical example for illustrative purposes only.**

As the correlation of the risk-mitigation portfolio with equities becomes more negative and the Sharpe ratio rises along with the number of strategies, the impact on the Sharpe ratio of a portfolio that combines an equity investment with an investment in the risk-mitigation portfolio increases. Like traditional portfolio theory, in which diversification is the only free lunch, combining strategies with positive return and negative correlation may be more akin to a free lunch at a three-star Michelin-rated restaurant: The correlation with equities becomes more negative and the risk-adjusted return becomes more positive as the number of strategies increases.

CAVEATS AND COMMENTS

Does this framework square with the standard capital asset pricing model (CAPM)? It certainly does not. CAPM states that asset expected returns should be proportional to asset betas. Any framework that assumes assets have positive return and negative beta is problematic in standard theory.

How to reconcile our perspective with theory?

One answer is that we are just mining the data; the asset betas that we derive from the data are noisy estimates that will prove meaningless. We would disagree with that view. Although sample betas can have large standard errors, we believe that the portfolio error is substantially lower due to averaging. We also know from looking at the asset universe that some pro-cyclical assets are expensive from a valuation perspective. This is another way of saying that a short position in these assets is an investment with negative beta and a positive expected Sharpe ratio.

Another answer is that an investment with a negative beta and a positive expected Sharpe ratio is a possibility but has exposure to other risk factors, as would be conjectured by arbitrage pricing theory. We have highlighted several well-known strategies that offer a negative equity beta and a positive return that hinge on exploiting other sources of return. Whether these sources are beta or alpha is an open question. Our conclusion: Investors looking for equity-risk-mitigation strategies may wish to use not one but a portfolio of such approaches.

APPENDIX 1: THE BETA OF A PUT

Posit the following random process for a non-dividend-paying asset price S:

$$\frac{dS}{S} = \mu dt + \sigma dW$$

where μ and σ are the drift and the volatility of the asset returns and dW is the increment of a Wiener process.

Then, by Itô's lemma, the infinitesimal relative change of a put price is given by

$$\frac{dP}{P} = \frac{\left(\frac{\partial P}{\partial S}\mu S + \frac{\partial P}{\partial t} + \frac{1}{2}\frac{\partial^2 P}{\partial S^2}\sigma^2 S^2\right)}{P}dt + \frac{\partial P}{\partial S}\frac{S}{P}\sigma dW.$$

The ratio of the put beta to the stock beta is

$$\frac{Cov\left(\frac{dP}{P},\frac{dS}{S}\right)}{Var(\frac{dS}{S})} = \frac{E\left(\frac{dP}{P}\frac{dS}{S}\right)}{E\left[\left(\frac{dS}{S}\right)^2\right]},$$

which boils down to

$$\frac{\frac{\partial P}{\partial S}\frac{S}{P}\sigma^2 dt}{\sigma^2 dt} = \frac{\partial P}{\partial S}\frac{S}{P},$$

that is, the put delta times the ratio of the underlying asset price to the put price.

APPENDIX 2: THE BETA OF AT-THE-MONEY FORWARD OPTIONS – AN APPROXIMATION

Recall that the value of a European call on a non-dividend-paying asset is

$$C = S\varphi\left\{\frac{ln\frac{S}{K} + (r + \frac{\sigma^2}{2})\tau}{\sigma\sqrt{\tau}}\right\} - Ke^{-r\tau}\varphi\left\{\frac{ln\frac{S}{K} + (r - \frac{\sigma^2}{2})\tau}{\sigma\sqrt{\tau}}\right\}$$

where K, r and τ are, respectively, the strike price, the riskless rate and the time to expiration.

For an ATMF option,

$$K = Se^{r\tau}.$$

Also, note that by put-call parity, an ATMF call price is equal to an ATMF put price.

From the call expression above, we get the simplified value of an ATMF put P:

$$\frac{P}{S} = \frac{C}{S} = \varphi\left(\frac{\sigma\sqrt{\tau}}{2}\right) - \varphi\left(\frac{-\sigma\sqrt{\tau}}{2}\right).$$

We can approximate this expression through a Taylor expansion:

$$\frac{P}{S} = \int_{\frac{-\sigma\sqrt{\tau}}{2}}^{\frac{\sigma\sqrt{\tau}}{2}}\frac{1}{\sqrt{2\pi}}e^{\frac{u^2}{2}}du \approx \int_{\frac{-\sigma\sqrt{\tau}}{2}}^{\frac{\sigma\sqrt{\tau}}{2}}\frac{1}{\sqrt{2\pi}}du + higher\ order\ terms = 0.4\,\sigma\sqrt{\tau}.$$

When $\tau = 1\ year$, $\frac{P}{S} = 0.4\sigma$ and $\frac{\partial P}{\partial S} = \varphi\left(\frac{\sigma}{2}\right) \approx \frac{0.4\sigma-1}{2}$, so that for an S&P 500 ATMF put:

$$\beta_{put} = \frac{S}{P}\frac{\partial P}{\partial S}\beta_{underlying} = -\frac{1.25}{\sigma} + \frac{1}{2}.$$

APPENDIX 3: CORRELATION OF THE AVERAGE OF TWO STRATEGIES IS MORE NEGATIVE THAN THE AVERAGE OF THE CORRELATIONS OF THE INDIVIDUAL STRATEGIES

Denote correlation between two random variables $\rho_{X,Z}$ as

$$\rho_{X,Z} = Corr(X,Z) = \frac{Cov(X,Z)}{\sigma_X\sigma_Z},$$

and the covariance of a linear combination of random variables is

$$Cov(X+Y,Z) = Cov(X,Z) + Cov(Y,Z),$$

so that the correlation can be written as

$$\rho_{X+Y,Z} = \frac{Cov(X,Z) + Cov(Y,Z)}{\sigma_{X+Y}\sigma_Z.}$$

Expanding this,

$$\rho_{X+Y,Z} = \frac{Cov(X,Z) + Cov(Y,Z)}{\sigma_Z\sqrt{\sigma_X^2 + \sigma_Y^2 + 2\rho_{X,Y}\sigma_X\sigma_Y}},$$

which can be rearranged as

$$\rho_{X+Y,Z} = \frac{\rho_{X,Z}\sigma_X + \rho_{Y,Z}\sigma_Y}{\sqrt{\sigma_X^2 + \sigma_Y^2 + 2\rho_{X,Y}\sigma_X\sigma_Y}}.$$

Let's suppose that $\sigma_X = \sigma_Y = \sigma$ are all equal to simplify (as in an equal-volatility-scaled portfolio)

$$\rho_{X+Y,Z} = \frac{(\rho_{X,Z} + \rho_{Y,Z})}{\sqrt{2(1 + \rho_{X,Y})}}.$$

Now assume that the average correlation of X with Z and Y with Z is denoted $\bar{\rho}_Z$:

$$\rho_{X+Y,Z} = \left(\frac{2}{\sqrt{2(1 + \rho_{X,Y})}}\right)\bar{\rho}_Z.$$

Then, as in our case where the correlation of our strategies with equities (Z) is $\bar{\rho}_Z < 0$, we would require that

$$2 > (1 + \rho_{X,Y}).$$

And because $\rho_{X,Y}$ is at most 1, it is always the case that a diversified group of negative beta strategies exhibits a stronger negative correlation than the average of the correlations.

APPENDIX 4: SHARPE RATIO EQUIVALENCE OF RISK-MITIGATION STRATEGY

The expected Sharpe ratio of a two-asset portfolio is

$$S = \frac{w_1\mu_1 + (1 - w_1)\mu_2}{\sqrt{w_1^2\sigma_1^2 + (1 - w_1)^2\sigma_2^2 + 2\rho_{1,2}w_1(1 - w_1)\sigma_1\sigma_2}}.$$

Suppose that asset 1 is equities. Then, in the case of a true "alternative" investment where the correlation with equities is zero, the Sharpe ratio of the portfolio simplifies to

$$S_p = \frac{w_1\mu_1 + (1 - w_1)\mu_2}{\sqrt{w_1^2\sigma_1^2 + (1 - w_1)^2\sigma_2^2}} \equiv \frac{\mu_p}{\sigma_p}.$$

Now suppose instead that we allocate to a portfolio of risk-mitigation strategies (asset 3) where the correlation is not zero but negative:

$$S(\rho_{1,3}|w_1,\mu_1,\mu_3,\sigma_1,\sigma_3) = \frac{w_1\mu_1 + (1 - w_1)\mu_3}{\sqrt{w_1^2\sigma_1^2 + (1 - w_1)^2\sigma_3^2 + 2\rho_{1,3}w_1(1 - w_1)\sigma_1\sigma_3}}.$$

Then, for a given correlation, weight in the nonequity investment, and volatility scale for the nonequity investment, we can derive the mean return on the risk-mitigation investment required so that the Sharpe ratios are equivalent. To do this, first define the required rate of return on the portfolio containing the alternative investment to deliver an equivalent Sharpe ratio of the portfolio invested in the risk-mitigation investment:

$$\mu_2 = \frac{S(\mu_3,\rho_{1,3}|w_1,\mu_1,\sigma_1,\sigma_3) * \sigma_p - w_1\mu_1}{1 - w_1}$$

Table 4A gives example values of μ_2/σ_2 – the required Sharpe ratio on the uncorrelated alternative investment – which is equivalent to the risk-mitigation strategy. Columns show the Sharpe ratio and rows represent equity correlation of the risk-mitigation strategy.

Table 4A: Required Sharpe ratio on alternative investment to be indifferent to risk-mitigation portfolio

	Sharpe ratio of risk-mitigation strategy				
	0.1	0.2	0.3	0.4	0.5
0	0.10	0.20	0.30	0.40	0.50
-0.1	0.13	0.24	0.34	0.44	0.54
-0.2	0.17	0.28	0.38	0.48	0.59
-0.3	0.21	0.32	0.42	0.53	0.63
-0.4	0.25	0.36	0.47	0.57	0.68
-0.5	0.29	0.40	0.51	0.62	0.73
-0.6	0.34	0.45	0.56	0.67	0.79
-0.7	0.39	0.50	0.62	0.73	0.84

Source: PIMCO

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Mr. Davis is an executive vice president and portfolio manager in the Newport Beach office. He is a senior member of PIMCO's quantitative strategies portfolio management team and manages multi-strategy alternatives, managed futures, managed volatility and tail risk hedging portfolios. He has conducted research and published extensively on systematic strategies, innovative hedging solutions and alternative risk premia. He began his career at PIMCO in the portfolio analytics group, where he worked on developing the core infrastructure for analyzing multi-asset portfolios and advised institutional clients globally. Prior to joining PIMCO in 2008, he was a consulting strategist with Prime International Trading in Chicago. He has 14 years of investment experience and holds a Ph.D. in economics with an emphasis on macroeconomics and finance from Northwestern University, where he also earned his master's degree. He holds undergraduate degrees in pure mathematics and management science from the University of California, San Diego.

Graham A. Rennison
Quantitative Portfolio Manager

Mr. Rennison is a senior vice president in the quantitative portfolio management group in the Newport Beach office, focusing on multi-asset-class systematic strategies. He was previously a member of the client analytics group, advising clients on strategic asset allocation. Prior to joining PIMCO in 2011, Mr. Rennison was a director and head of systematic strategies research at Barclays Capital in New York and also spent five years at Lehman Brothers. He has 15 years of investment experience and holds master's and undergraduate degrees in mathematics from Cambridge University, England

[1] The taper tantrum of 2013 coincided with an abrupt and violent shift in correlations, which coincided with large short-term losses in fixed income assets.

[2] https://www.newyorkfed.org/research/data_indicators/term_premia.html
  The authors propose a regression-based approach to the pricing of interest rates. "Pricing the Term Structure with Linear Regressions" by Tobias Adrian, Richard K. Crump and Emanuel Moench, *Journal of Financial Economics 110,* no. 1 (October 2013): 110-38.

[3] The data is obtained from the Bloomberg subindexes for each of the listed commodities.

[4] The term $P_{X,Y}$ is in the denominator. The closer it is to zero, the more the correlation of the portfolio of strategies is magnified versus the average correlation of the strategies.

The **"risk-free rate"** can be considered the return on an investment that, in theory, carries no risk. Therefore, it is implied that any additional risk should be rewarded with additional return. All investments contain risk and may lose value.

Past performance is not a guarantee or a reliable indicator of future results.

The analysis contained in this paper is based on hypothetical modeling. No representation is being made that any account, product, or strategy will or is likely to achieve profits, losses, or results similar to those shown. Hypothetical or simulated performance results have several inherent limitations. Unlike an actual performance record, simulated results do not represent actual performance and are generally prepared with the benefit of hindsight. There are frequently sharp differences between simulated performance results and the actual results subsequently achieved by any particular account, product, or strategy. In addition, since trades have not actually been executed, simulated results cannot account for the impact of certain market risks such as lack of liquidity. There are numerous other factors related to the markets in general or the implementation of any specific investment strategy, which cannot be fully accounted for in the preparation of simulated results and all of which can adversely affect actual results.

Details of the Basic Trend-Follower: For the purposes of this analysis we set up a simple, transparent hypothetical trend-following strategy. The strategy trades twenty markets: five each in equity index, bond, currency and commodity futures. The strategy trades once per week, taking a long position if the current futures price is above the one year moving average price, and taking a short position if it is below. Each position is scaled inversely to the recent 3-month daily realized volatility of the contract, and the overall strategy is scaled to target 10% volatility using trailing 10-year windows to estimate volatility of the strategy. Some futures markets were unavailable in the early parts of the sample. In those periods, risk allocated to each asset class is kept roughly constant over long periods of time by scaling up the underrepresented sectors. Over short periods, risk can be skewed to some asset classes. Fixed transaction costs estimated from available market data for each futures market, of between 1bp and 10bps, are subtracted from returns. For historical data before 1987, extended hypothetical futures time series are constructed for S&P 500, five-year note futures and currency futures (JPY, DEM, AUD, GBP) before actual trading in those futures markets began. For S&P 500 futures we use daily excess return data from the Ken French database for the top 30% of U.S. stocks with reinvested dividends. For five-year note futures we use the Gurkaynak, Sack, Wright constant maturity Treasury yield data set to estimate daily returns, including roll-down and carry. Delivery option effects are not included in the modelling but would not be expected to bias results. Proxy currency future returns are calculated using risk-free rate data from Dimson, Marsh and Staunton and Bloomberg spot rates, starting in 1973. A useful comparison is with the SG Trend Index, a composite of the top 10 trend-following hedge funds. In order to make a meaningful comparison we add to the returns of the Basic Trend Follower a proxy for collateral returns (we use the Barclays Short Term Treasury index total returns) and adjust for assumed fees equal 2% running plus 20% of gains assessed annually. Annual correlation of these two series is 64% over this period. Average annual total returns are similar at 5.2% and 6.3% for the basic trend follower and SG Index respectively.

Return assumptions are for illustrative purposes only and are not a prediction or a projection of return. Return assumption is an estimate of what investments may earn on average over a 10 year period. Actual returns may be higher or lower than those shown and may vary substantially over shorter time periods. Return assumptions are subject to change without notice.

Figures are provided for illustrative purposes and are not indicative of the past or future performance of any PIMCO product.

All investments contain risk and may lose value. Investing in the **bond market** is subject to risks, including market, interest rate, issuer, credit, inflation risk, and liquidity risk. The value of most bonds and bond strategies are impacted by changes in interest rates. Bonds and bond strategies with longer durations tend to be more sensitive and volatile than those with shorter durations; bond prices generally fall as interest rates rise, and the current low interest rate environment increases this risk. Current reductions in bond counterparty capacity may contribute to decreased market liquidity and increased price volatility. Bond investments may be worth more or less than the original cost when redeemed. **Equities** may decline in value due to both real and perceived general market, economic and industry conditions. **Commodities** contain heightened risk, including market, political, regulatory and natural conditions, and may not be suitable for all investors. **Tail risk hedging** may involve entering into financial derivatives that are expected to increase in value during the occurrence of tail events. Investing in a tail event instrument could lose all or a portion of its value even in a period of severe market stress. A tail event is unpredictable; therefore, investments in instruments tied to the occurrence of a tail event are speculative. **Derivatives** may involve certain costs and risks such as liquidity, interest rate, market, credit, management and the risk that a position could not be closed when most advantageous. Investing in derivatives could lose more than the amount invested. The **use of models** to evaluate securities or securities markets based on certain assumptions concerning the interplay of market factors, may not adequately take into account certain factors, may not perform as intended, and may result in a decline in the value of an investment, which could be substantial. Investors should **consult their investment professional** prior to making an investment decision.

The **SG Trend Index** calculates the net daily rate of return for a group of 10 trend following CTAs selected from the largest managers open to new investment. The SG Trend Index is equal-weighted and reconstituted annually and has become recognized as the key managed futures trend following performance benchmark. The **S&P 500 Index** is an unmanaged market index generally considered representative of the stock market as a whole. The index focuses on the Large-Cap segment of the U.S. equities market. The **MSCI World Index** is a free float-adjusted market capitalization weighted index that is designed to measure the equity market performance of developed markets. The MSCI World Index consists of the following 24 developed market country indices: Australia, Austria, Belgium, Canada, Denmark, Finland, France, Germany, Greece, Hong Kong, Ireland, Israel, Italy, Japan, Netherlands, New Zealand, Norway, Portugal, Singapore, Spain, Sweden, Switzerland, the United Kingdom, and the United States. The **RAE Low Volatility International model portfolio** contains stocks of large Developed Markets ex-U.S. companies ranked by the Fundamental Index® methodology and further screened by a composite score of yield, volatility, and leverage. Companies are weighted by the product of the fundamental score of each company and a factor that reduces the weight for higher beta companies (and increases those for lower beta companies). The portfolio is rebalanced on a monthly staggered basis. Companies in 23 developed ex US market countries are eligible for inclusion. Prior to 15 April 2015, this benchmark was known as the International RA Low Volatility Equity Income. The **RAE Low Volatility US model portfolio** contains stocks of large U.S.-listed companies ranked by the Fundamental Index® methodology and further screened by a composite score of yield, volatility, and leverage. Companies are weighted by the product of the fundamental score of each company and a factor that reduces the weight for higher beta companies (and increases those for lower beta companies). The portfolio is rebalanced on a monthly staggered basis. Prior to 15 April 2015, this benchmark was known as the US RA Low Volatility Equity Income. It is not possible to invest directly in an unmanaged index.

Alpha is a measure of performance on a risk-adjusted basis calculated by comparing the volatility (price risk) of a portfolio vs. its risk-adjusted performance to a benchmark index; the excess return relative to the benchmark is alpha. **Beta** is a measure of price sensitivity to market movements. Market beta is 1. **Correlation** is a statistical measure of how two securities move in relation to each other.

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