Opinions in the active-passive investment debate have drifted poles apart over recent years. We revisit this discussion by contrasting equity and fixed income markets in the U.S. We look at performance numbers and find that, unlike their stock counterparts, active bond mutual funds and exchange-traded funds (ETFs) have largely outperformed their passive peers after fees. We offer conjectures as to why bonds are different from stocks. This may be due to the large proportion of noneconomic bond investors, the benchmark rebalancing frequency and turnover, structural tilts in fixed income space, the wide range of financial derivatives available to active bond managers, and security-level credit research and new issue concessions. At a macro level, we believe that a purely passive market would cause severe market risk and resource misallocations. Realistically, neither passive nor active investors can fully dominate at equilibrium. Of course, passive management has its virtues. Yet there is reason to believe that, unchecked, passive management may encourage free riding, adverse selection and moral hazard.
POINT #1 – ACTIVE BOND FUNDS AND ETFS LARGELY OUTPERFORMED THEIR MEDIAN PASSIVE PEERS

Despite the general presumption of underperformance, more than half of the active bond mutual funds and ETFs beat their median passive peers in most categories over the past 1, 3, 5, 7 and 10 years, with 63% of them outperforming over the past 5 years. In contrast, only 43% of active equity mutual funds and ETFs outperformed their median passive peers over the past 5 years. Taking the three largest categories within fixed income for the same 5-year period, 84%, 81% and 60% of active funds and ETFs outperformed their median passive peers in intermediate-term, high yield and short-term categories, respectively. Within equity, most active funds and ETFs in each of the three largest categories – large growth, large blend and large value – underperformed (see Figure 1).

These results should come with one caveat. The fact that active bond funds and ETFs outperformed passive does not necessarily mean all active outperformed passive. Active bond managers also include non-indexed investors such as central banks, commercial banks, large parts of the insurance industry and retail. Some of these investors operate under tight constraints which may affect their performance (more on this in Point #4).

POINT #2 – ACTIVE BOND FUNDS AND ETFS LARGELY OUTPERFORMED THEIR BENCHMARKS EXCEPT WHEN THE COST OF BENCHMARK REPLICATION WAS PROHIBITIVE

Figure 2 shows the percentages of active mutual funds and ETFs that outperformed their primary prospectus benchmarks after fees for the past 1, 3, 5, 7 and 10 years; it shows the aggregated results for broad bond and equity groups and each of the three largest categories within bonds and equity.

Again, more than half of the active bond mutual funds and ETFs beat their benchmarks in most categories over the past 1, 3, 5 and 7 years, with 61% of them outperforming over the past 5 years. This stands in strong contrast with equity results, where only 35% of active funds and ETFs outperformed their benchmarks over the past 5 years. Taking the three largest categories within fixed income for the same 5-year period, 82% and 84% of active funds and ETFs in the Intermediate-Term Bond and Short-Term Bond categories outperformed their benchmarks while only 25% in the High Yield Bond category outperformed. Within equity, most active funds and ETFs in each of the three largest categories – large growth, large blend and large value – underperformed.

Figure 1: Percentage of active mutual funds and ETFs that outperformed their median passive peers after fees

Source: Morningstar Direct as of 31 December 2016. Past performance is not a guarantee or a reliable indicator of future results. The three largest categories are based on numbers of active mutual funds and ETFs with at least one-year return histories. Based on Morningstar U.S. ETF and U.S. Open-End Fund categories (institutional shares only). To avoid potential survivorship bias, we included funds and ETFs that were live at the beginning of each sample period but were liquidated or merged as of 31 December 2016. For the High Yield Bond and Short-Term Bond categories, 10-year outperformance numbers are not available due to the lack of passive peer groups. Chart is provided for illustrative purposes and is not indicative of the past or future performance of any PIMCO product.
Note that active bond funds and ETFs may underperform their benchmarks but still outperform median passive peers. Consider the High Yield Bond category. Although the percentage of active funds and ETFs outperforming their benchmarks for this category appears low, 81% outperformed their median passive peers over the same 5-year period. This indicates the difficulty of replicating the performance of high yield benchmarks, possibly due to the lower liquidity of the market and the high transaction costs for all but the largest issues.

The 10-year period started with what is considered by many to be the worst global financial crisis since the 1930s. During this period, market liquidity dried up and bid-ask spreads widened, dramatically increasing trading costs for both active and passive investors. Because benchmark returns do not reflect these frictional costs, it is more challenging to beat the benchmarks, especially in the relatively illiquid and inefficient market. This may explain the underperformance of active bond funds versus their benchmarks over the 10-year period.

Figure 3 summarizes the two types of outperformance measures for broad bonds and equity groups. The percentages of active bond funds and ETFs outperforming their benchmarks and those outperforming their median passive peers over the past 1, 3, 5 and 7 years all exceeded 50%; more than half outperformed their median passive peers over the past 10 years. In contrast, the percentages for active equity funds and ETFs for both measures and over all time periods considered were less than 50%.

Figure 3: Outperformance over benchmarks and outperformance over median passive peers after fees

Source: Morningstar Direct as of 31 December 2016. Past performance is not a guarantee or a reliable indicator of future results. Based on Morningstar U.S. ETF and U.S. Open-End Fund categories (institutional shares only). To avoid potential survivorship bias, we included funds and ETFs that were live at the beginning of each sample period but were liquidated or merged as of 31 December 2016. Some categories contain funds with a wide range of benchmarks. Chart is provided for illustrative purposes and is not indicative of the past or future performance of any PIMCO product.

Figure 2: Percentage of active mutual funds and ETFs that outperformed their primary prospectus benchmarks after fees

Source: Morningstar Direct as of 31 December 2016. Past performance is not a guarantee or a reliable indicator of future results. The three largest categories are based on numbers of active mutual funds and ETFs with at least one-year return histories. Based on Morningstar U.S. ETF and U.S. Open-End Fund categories (institutional shares only). To avoid potential survivorship bias, we included active funds and ETFs that were live at the beginning of each sample period but were liquidated or merged as of 31 December 2016. Some categories contain funds with a wide range of benchmarks. Chart is provided for illustrative purposes and is not indicative of the past or future performance of any PIMCO product.
POINT #3 – ACTIVE MANAGERS SHOULD BE EVALUATED OVER A LONG HORIZON

It is important for bond fund managers to show some healthy skepticism for the performance data – not only because the data could be self-serving, but also because they are inherently noisy.

To illustrate this point, consider the following thought experiment: Suppose that a skilled hypothetical portfolio manager has an information ratio of 0.5. The volatility of the active portfolio is 4.0% and that of the index is 3.3%. The correlation of the portfolio return with the index return is 0.9. Note that all these numbers would be empirically plausible for an active bond manager in the top quartile ranked by risk-adjusted alphas. The question is, how long would it take for the hypothetical active manager to beat the index with a 90% probability? The answer, it turns out, is 7 years. (We show the mathematics behind this example in Appendix 1).

What does it all mean? First, this example illustrates the reality of a low signal-to-noise ratio in bond asset management. Given the higher tracking error and lower information ratios displayed in manager performance, it is even lower for equities. Second, it shows that managers are typically evaluated over too short a time frame: Over 1 year, this hypothetical manager would have a 69% probability of outperforming and over 3 years, the probability would rise to 80%. And third, the answer is very sensitive to assumptions. Table 1 illustrates the relationship between the time it takes to outperform the index with 90% confidence and the information ratio under the volatility and correlation assumptions described above.

Table 1: Number of years needed for the manager to beat the index with 90% confidence

<table>
<thead>
<tr>
<th>Information ratio</th>
<th>Years</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.7</td>
<td>3.5</td>
</tr>
<tr>
<td>0.5</td>
<td>7.0</td>
</tr>
<tr>
<td>0.3</td>
<td>20.0</td>
</tr>
<tr>
<td>0.2</td>
<td>48.0</td>
</tr>
</tbody>
</table>

Source: PIMCO. Hypothetical example for illustrative purposes only. Table is provided for illustrative purposes and is not indicative of the past or future performance of any PIMCO product.

If we fix the horizon at 5 years with the same risk assumptions, it appears that the probability of outperformance is an increasing and concave function of the information ratio, as one would expect (see Figure 4).

Figure 4: Probability of outperformance in 5 years

POINT #4: IT IS NOT JUST ABOUT ACTIVE VERSUS PASSIVE

The straw man in the active-passive debate stems from a short paper by William Sharpe titled “The Arithmetic of Active Management”: If passive is defined as holding the market portfolio, and active is everything not passive, then it trivially follows that active managers as a whole, like passive managers, hold the market portfolio at any time. Both groups will therefore generate the same returns before fees. To the extent that active fees are higher than passive fees, passive managers will outperform active managers, on average. Plain and simple, but maybe too much so. It is difficult not to be reminded of Einstein’s quote: “Everything should be made as simple as possible, but not simpler.”

As appealing as Sharpe’s arithmetic is, the deficits of the argument (some of them acknowledged by Sharpe himself) are obvious. To start with, the distinction between active and passive is wanting in bond space. At a minimum, one can divide the investors’ universe into three categories: passive, economic and noneconomic. This is because noneconomic investors are plentiful in fixed income (see Table 2): Central banks buy bonds to depreciate their currency and boost inflation, growth and asset prices; commercial banks and insurance companies may care more about book yield than total return for a variety of reasons, such as accounting rules, other regulations or a simple preference for predictable, low-turnover portfolios. It is worth
noting that these investors make up a meaningful portion of global bond buyers – roughly 47% of the $102 trillion global bond market. Similarly, because of their guidelines, both noneconomic constrained investors and passive index buyers may be forced to sell all bonds that fall below the investment grade threshold or be late to the game buying bonds whose upgrade is foreseeable because they must wait for a ratings agency imprimatur. The list goes on and on.

To the extent constraints are binding (most of them are), by construction, economic investors tend to outperform noneconomic investors, as the former buy cheap fallen angels from the latter and sell them expensive high-coupon bonds. Active managers potentially may also be compensated by passive managers for providing them with liquidity around changes in index construction. So while active and passive managers may in theory generate the same returns before fees, the further categorization of investors into three groups illustrates that economic investors may outperform passive investors which, in turn, are likely to outperform noneconomic investors before fees.

Table 2: Bond holdings by noneconomic investors

<table>
<thead>
<tr>
<th>Investor group</th>
<th>Bond holdings ($ trillion)</th>
<th>Investment objective</th>
</tr>
</thead>
<tbody>
<tr>
<td>Central banks</td>
<td></td>
<td></td>
</tr>
<tr>
<td>– Foreign exchange</td>
<td>10.8</td>
<td>Stabilize exchange rates</td>
</tr>
<tr>
<td>– Reserves</td>
<td></td>
<td></td>
</tr>
<tr>
<td>– Domestic holdings</td>
<td>4.5</td>
<td>Manage money supply</td>
</tr>
<tr>
<td>U.S. insurance</td>
<td>4.3</td>
<td>Book yield, predictable income, regulatory-driven capital charges</td>
</tr>
<tr>
<td>U.S. banks</td>
<td>2.8</td>
<td></td>
</tr>
<tr>
<td>European insurers</td>
<td>5.3</td>
<td></td>
</tr>
<tr>
<td>European banks</td>
<td>4.7</td>
<td></td>
</tr>
<tr>
<td>Asian banks and insurers</td>
<td>12.6</td>
<td></td>
</tr>
<tr>
<td>Other banks and insurers</td>
<td>2.0 – 3.0</td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td>47.0 – 48.0</td>
<td></td>
</tr>
</tbody>
</table>

Source: Company filings, European Federation, EIOPA, EBA, SNL Financial, Bloomberg and PIMCO. As of 31 December 2016.

There are other assumptions underlying Sharpe's arithmetic that, when violated in reality, may give rise to active management opportunities over passive. For example, Sharpe's work assumes investors have the same investment horizon, risk aversion and tax rate.

### POINT #5 – INFORMATION IS VITAL TO TRADING AND REBALANCING

Sharpe's arithmetic implicitly assumes passive investors buy and hold and don't trade securities. In reality, most bond indexes are rebalanced monthly, requiring both active and passive investors to trade, if only because bonds mature, new bonds are issued, and index inclusion and exclusion rules create movement in and out of the index (see Pedersen (2017) for similar arguments for equity). The average turnover rate for the Bloomberg Barclays US Aggregate Bond Index for the past 3 years (as of 31 December 2016) was about 40% per year, half of which was due to new issues of securities. To the extent they do not trade pure noise, active managers seek to be better informed than passive managers. They invest in fixed income strategists, quants, credit analysts and systems to acquire and process relevant information and make better investment decisions. Because their cost of being informed is higher, active managers will typically command higher fees. They also will tend to use their knowledge to seek securities they can buy at a lower offer and sell at a higher bid than passive managers. Given that new securities make up about 20% of the bond market's capitalization every year, a strong active presence in the new bond issuance market has the potential to materially add value for fixed income investors by identifying those issues with the most attractive valuations. Additionally, active bond managers generally strive to secure sizable allocations at concession in the process of syndication of new issues. Last but not least, security-level credit research can provide a decisive advantage in the bond investment process (Worah and Mattu, 2014).

In contrast, most equity indexes are rebalanced annually or quarterly. The annual turnover rate of the S&P 500 index was about 4% for the past year. New issues are generally less than 1% of the market capitalization given that common stocks are generally perpetual securities. Therefore, the contribution of a strong presence in the new issuance market to performance for equity would be much less significant compared with that for bonds. Mauboussin et al. (2017) show a sharp fall in the number of listed stocks in the U.S. since 1996 due to rising listing costs, including higher costs for greater information disclosure. Consequently, listed companies today are bigger, older and better established than they were two decades ago. Mauboussin et al. speculate this trend has contributed to greater informational efficiency and fewer opportunities with material mispricing in the U.S. stock market.
In summary, the higher turnover in bond indexes and lower informational efficiency of bond markets help explain why active bond funds have outperformed more often over our sample period than their equity counterparts.

**POINT #6 – OFF-INDEX AND ACTIVE SHARE MATTER**

Structural tilts, off-index or otherwise, are bond managers’ staples in the battle against the index. Duration, yield curve steepeners, high yielding currencies, high yield credit spreads, agency and non agency mortgage spreads, volatility sales and liquidity premia – to name a few – are structural tilts that can be an important source of durable added value.

To show that structural tilts are more than urban myths, we correlate excess returns of active bond funds with excess returns from specific factors, specifically duration, investment grade and high yield spreads (see Table 3). The picture that emerges is clear: Active bond funds and ETFs in the largest taxable bond category, Intermediate-Term Bond, are structurally short duration, long investment grade and long high yield exposure against the index (either directly or indirectly through factor tilts not included in the analysis). Regressing excess return against factors (see Appendix 2) also shows that exploiting credit and liquidity risk premia is a particularly important ingredient in the (not so secret) sauce. Adjusted t-statistics and R-squared show that tilts have been central to portfolios beyond a reasonable doubt.

As is well known, a number of structural tilts are off-index: High yield, as just mentioned, as well as emerging market bonds and currencies, are fertile ground for outperformance opportunities yet are not part of typical bond indexes. This may be another reason active funds have generally done well in fixed income over the sample period. Of course, passive investors can add emerging market and high yield ETFs to their portfolios, but the lack of granularity and the opacity of such ETFs can be problematic, making it difficult to mix and match.

There is a wide range of financial derivatives available to the active bond managers that allow for potentially profitable expressions of investment themes: currency swap basis, futures basis, CDS-cash basis and TBA rolls are such examples. In addition, active bond managers could implement so-called smart strategies such as carry, value and momentum, which have historically displayed substantially positive Sharpe ratios (see, for example, Baz et al., 2015).

A related point is that active share may matter in bond management, meaning that the more portfolio positions differ from the index, the more potential that the fund will outperform. Because we do not have direct access to fund managers’ positions, we use the correlation between portfolio returns and index returns as a proxy for the degree of “index hugging” (the higher the correlation, the lower the active share). The correlation between excess returns and portfolio-index return correlations is -0.57 for the past 5 years (see Figure 5). So it may pay to deviate. Or, to put it differently, when an active manager deviates, this may indicate the extent of potentially profitable investment ideas in the portfolio.

<table>
<thead>
<tr>
<th></th>
<th>Mean (annualized)</th>
<th>Volatility (annualized)</th>
<th>Mean ER</th>
<th>Dur</th>
<th>IG</th>
<th>HY</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean ER</td>
<td>0.4%</td>
<td>1.9%</td>
<td>1.00</td>
<td>-0.52</td>
<td>0.81</td>
<td>0.89</td>
</tr>
<tr>
<td>Dur</td>
<td>0.6%</td>
<td>0.8%</td>
<td>-0.52</td>
<td>1.00</td>
<td>-0.40</td>
<td>-0.49</td>
</tr>
<tr>
<td>IG</td>
<td>0.1%</td>
<td>1.0%</td>
<td>0.81</td>
<td>-0.40</td>
<td>1.00</td>
<td>0.87</td>
</tr>
<tr>
<td>HY</td>
<td>1.0%</td>
<td>2.9%</td>
<td>0.89</td>
<td>-0.49</td>
<td>0.87</td>
<td>1.00</td>
</tr>
</tbody>
</table>

Source: Morningstar and Barclays as of 31 December 2016. Duration: Excess returns of Bloomberg Barclays US Treasury Index over cash (1M OIS), per unit of duration. Investment grade: Excess returns of Bloomberg Barclays US Corporate IG Index over duration-matched Treasuries, per unit of spread duration. High yield: Excess returns of Bloomberg Barclays US Corporate HY Index over duration-matched Treasuries, per unit of spread duration. Past performance is not a guarantee or a reliable indicator of future results. Sample consists of average monthly excess returns for active mutual funds and ETFs in “Intermediate-Term Bond” Morningstar category, with Bloomberg Barclays US Aggregate Bond Index as their primary prospectus benchmark and at least 10-year return histories. Table is provided for illustrative purposes and is not indicative of the past or future performance of any PIMCO product.
Figure 5: A scatter plot of 5-year excess returns against fund-index return correlations

POINT #7 – STRUCTURAL TILTS ARE NOT ALWAYS BETA

We are then left with the obvious question: Is the outperformance alpha or beta? On this point, our indecision is final. Beta, of course, refers to systematic risk factors that need to be remunerated at equilibrium, whereas alpha is defined as residual return in an asset pricing equation and is associated with investing and trading skills.

We are skeptical about this alpha-beta debate because systematic factors, as commonly understood today by both academics and practitioners, have weak theoretical underpinnings. How have some trading strategies graduated to systematic factor status? In other words, what is a systematic factor? A realistic answer may be: whatever has worked in the U.S. over the past few decades. Why is “duration” a systematic factor rather than sunspots? Because duration extension has worked over the past 30 years, as opposed to sunspots. And duration has worked because monetary policy has been accommodative and has allowed large returns from carry, roll-downs and capital gains in bond markets. In other words, betas may well be an artifact of the data sample and the ultimate exercise in data mining (Harvey et al., 2015). To come back to the “systematic duration factor,” it has been tested over a period that may well be statistically insignificant because it boiled down to a single monetary experiment.

If all this were true, then systematic factor tilts are less beta and more alpha than investors are prone to think. As to the timing of those tilts, there is a strong presumption that they are alpha.

POINT #8 – A PURELY PASSIVE MARKET WOULD CAUSE SEVERE MARKET RISK AND RESOURCE MISALLOCATIONS

What would financial markets, including equities, look like in a world where asset management is purely passive? Not very compelling, it turns out, and for a reason: In a world where every asset manager is passive, the asset management mandate is to replicate the market. Therefore, all assets get absorbed without due consideration of their characteristics – cash flows, governance and broad risk/return parameters. Prices would cease to be informative the day assets got bought without being analyzed. The market would be subject to a degenerate form of Say’s law, under which supply creates its own demand, yet suppliers of securities – bond and equity issuers – are better informed about these securities than are purchasers, in this case passive investors. One can easily see how the market equilibrium would become inherently expensive as passive price takers kept buying everything on supply and correlation across securities increased. This would, of course, lead to capital misallocation on a grand scale (with money chasing expensive assets), to a market crash of sorts and to a collapse in confidence.

POINT #9 – NEITHER PASSIVE NOR ACTIVE INVESTORS CAN DOMINATE AT EQUILIBRIUM

Thankfully, it is hard to see how asset management turns 100% passive. In a classic 1980 paper, Grossman and Stiglitz argue that markets cannot be informationally efficient, meaning that prices cannot perfectly reflect available information. Why can't they? Because if they do, there is no incentive for anyone to acquire and process the information, in which case there is no reason to trade and the market becomes passive. But as new information infiltrates this market, discrepancies arise and it becomes profitable to acquire information and trade the market. Clearly, investors have a stronger incentive to become active when most investors go passive, and vice versa. In a world with noneconomic investors, the math gets more complicated but the cost of on-demand liquidity almost certainly goes up, allowing more rents for economic investors. The pendulum will swing as either the passive or the active population dominates the market.
All it takes is a small proportion of active informed investors to ensure market prices are informative. These investors would be the marginal price setters while passive investors would enjoy a piggyback ride. In this context, information derived from prices is a public good. Active management, then, is a public good as well. And an absence thereof, as outlined above, can lead to undesirable outcomes.

Not only may passive management be guilty of free riding, it may also lead to adverse selection and moral hazard. To the extent that index inclusion rules favor large borrowers in bond space, passive investment, by channeling funds into indexes, results in excess demand for large borrowers’ debt. This is a clear case of misdirecting capital toward those who deserve it the least, which is sometimes called the Matthew effect: “For he that hath, to him shall be given: and from him that has not shall be taken even that which he hath.” Furthermore, by causing higher prices and lower spreads on the debt of relatively profligate borrowers, a flurry of moral hazards emerge: Governments borrowing at artificially low spreads postpone needed reforms, corporations that overborrow misuse the cash, and so on.

Although on the surface low-fee passive vehicles may benefit savers and pensioners, the reality is more nuanced, to put it mildly. In economic parlance, passive investment produces a host of negative externalities. These grow, likely in a non linear fashion, as the fraction invested passively rises materially. A disturbing implication, well known to economists, is that negative externalities ought to be taxed and positive externalities subsidized – all topics beyond the scope of this paper.

Passive managers’ goal is to replicate the performance of a market index by holding the same securities or a sampling of the securities in the index. They do not need to spend resources to beat the benchmarks. As a result, they usually charge much lower fees than active managers. For many investors who want to focus on other investment decisions, such as asset allocation, and only seek index replication at the asset class level, passive investment provides a cost-effective way to access individual markets.

Active managers play an important role in the economy by helping to allocate capital efficiently. However, there will be times when they overinvest in research and information acquisition in certain markets or market segments. Having a healthy number of passive choices in each market helps keep this in check. Because most indexes are not directly investable, the competition between active and passive managers will allow investors to screen out active managers that charge higher fees without adding value relative to their passive peers. Cremers et al. (2016) find actively managed mutual funds are more active, charge lower fees and generate higher alpha when they face more competitive pressure from low-cost explicitly indexed funds.

The optimal mix of active and passive investment should depend on the market. A highly efficient and liquid market where active managers’ investment in research and information acquisition can no longer be adequately compensated by superior performance over passive, on average, may benefit from less active and more passive management.
Since active management is inevitable, and given the outperformance demonstrated over our sample period, why all the fuss over active management? After all, there is no such thing as passive, just different shades of active management.

**CONCLUSIONS**

Opinions in the active-passive investment debate have drifted poles apart over recent years. We revisit this discussion by contrasting equity and bonds. We look at performance numbers and find that, unlike their stock counterparts, active bond mutual funds have largely outperformed their median passive peers over our sample period. We offer conjectures as to why bonds and stocks differ.

This may be due to:

- the large proportion of noneconomic bond investors
- benchmark rebalancing frequency and turnover
- structural tilts in fixed income space
- the wide range of financial derivatives available to active bond managers
- security-level credit research and new issue concessions

At a macro level, we believe that a purely passive market would cause severe market risk and resource misallocations. Realistically, neither passive nor active investors can fully dominate at equilibrium. Of course, passive management has its virtues. Yet there is reason to believe that, unchecked, passive management may encourage free riding, adverse selection and moral hazard.

We thank Markus Aakko, Mark Anson, Charles de Segundo, Chris Dialynas, Mohsen Fahmi, Sudi Mariappa, Vasant Naik, Rama Nambimadom, Emmanuel Roman, Steve Sapra, Lutz Schloegl, Emmanuel Sharef, and Mihir Worah for valuable comments on earlier versions of the manuscript, and Robert Selouan for data support.
**APPENDIX 1: SIGNAL VERSUS NOISE**

Consider a money manager whose portfolio value $P$ follows a geometric Brownian motion:

$$\frac{dP}{P} = \mu_P dt + \sigma_P dW_P.$$ 

The money manager is trying to beat an index that also follows a geometric Brownian motion:

$$\frac{dI}{I} = \mu_I dt + \sigma_I dW_I,$$

$dW_P$ and $dW_I$ have correlation $\rho$. The time unit is 1 year.

We can state the values of the processes at time $t$:

$$P_t = P_0 \exp \left[ \left( \mu_P - \frac{1}{2} \sigma_P^2 \right) t + \sigma_P \sqrt{t} Z_P \right],$$

$$I_t = I_0 \exp \left[ \left( \mu_I - \frac{1}{2} \sigma_I^2 \right) t + \sigma_I \sqrt{t} Z_I \right].$$

$Z_P$ and $Z_I$ are two standard normal random variables with correlation $\rho$.

The probability that the manager beats the index at time $t$:

$$P \left( \frac{P_t}{P_0} > \frac{I_t}{I_0} \right) = P \left( \left( \mu_P - \frac{1}{2} \sigma_P^2 \right) t + \sigma_P \sqrt{t} Z_P > \left( \mu_I - \frac{1}{2} \sigma_I^2 \right) t + \sigma_I \sqrt{t} Z_I \right)$$

$$= P \left( \frac{\sigma_P Z_P - \sigma_I Z_I}{\sqrt{\sigma_P^2 + \sigma_I^2 - 2\rho \sigma_P \sigma_I}} > \frac{\left( \mu_P - \frac{1}{2} \sigma_P^2 \right) - \left( \mu_I - \frac{1}{2} \sigma_I^2 \right)}{\sqrt{\sigma_P^2 + \sigma_I^2 - 2\rho \sigma_P \sigma_I}} \sqrt{t} \right)$$

$$= \Phi \left( \frac{\left( \mu_P - \frac{1}{2} \sigma_P^2 \right) - \left( \mu_I - \frac{1}{2} \sigma_I^2 \right)}{\sqrt{\sigma_P^2 + \sigma_I^2 - 2\rho \sigma_P \sigma_I}} \sqrt{t} \right)$$

where $\Phi$ is the cumulative distribution function (CDF) of a standard normal random variable. Define the information ratio as the ratio between annualized alpha and tracking error, i.e., $\frac{\mu_P - \mu_I}{\sqrt{\sigma_P^2 + \sigma_I^2 - 2\rho \sigma_P \sigma_I}}$. The probability of outperformance can be estimated given assumptions on the horizon, the information ratio, and the volatility and correlation parameters.

See Ambarish and Siegel (1996) for a more detailed discussion.

**APPENDIX 2: SUMMARY OF FUND-SPECIFIC REGRESSIONS OF MONTHLY ALPHAS ON RISK FACTOR RETURNS BY ALPHA QUARTILE (JANUARY 2007–DECEMBER 2016)**

<table>
<thead>
<tr>
<th>Quartiles</th>
<th>Average annualized alpha</th>
<th>Constant</th>
<th>Dur</th>
<th>IG</th>
<th>HY</th>
<th>Constant</th>
<th>Dur</th>
<th>IG</th>
<th>HY</th>
<th>Average Adj R2</th>
</tr>
</thead>
<tbody>
<tr>
<td>1st</td>
<td>-0.45%</td>
<td>-0.06%</td>
<td>-0.41</td>
<td>0.05</td>
<td>0.44</td>
<td>-1.29</td>
<td>-1.11</td>
<td>0.46</td>
<td>2.67</td>
<td>48%</td>
</tr>
<tr>
<td>2nd</td>
<td>0.27%</td>
<td>0.01%</td>
<td>-0.25</td>
<td>0.22</td>
<td>0.29</td>
<td>0.32</td>
<td>-0.71</td>
<td>0.97</td>
<td>2.56</td>
<td>46%</td>
</tr>
<tr>
<td>3rd</td>
<td>0.56%</td>
<td>0.01%</td>
<td>-0.14</td>
<td>0.51</td>
<td>0.45</td>
<td>0.34</td>
<td>-0.35</td>
<td>1.52</td>
<td>3.51</td>
<td>57%</td>
</tr>
<tr>
<td>4th</td>
<td>1.19%</td>
<td>0.05%</td>
<td>-0.24</td>
<td>0.45</td>
<td>0.60</td>
<td>1.17</td>
<td>-0.53</td>
<td>1.05</td>
<td>3.18</td>
<td>51%</td>
</tr>
</tbody>
</table>

Source: Morningstar and Barclays as of 31 December 2016.
Duration: Excess returns of Bloomberg Barclays US Treasury Index over cash (1M OIS), per unit of duration.
Investment grade: Excess returns of Bloomberg Barclays US Corporate IG Index over duration-matched Treasuries, per unit of spread duration.
High yield: Excess returns of Bloomberg Barclays US Corporate HY Index over duration-matched Treasuries, per unit of spread duration.
Quartiles are based on 10-year annualized alphas. Newey-West heteroskedasticity and autocorrelation consistent t-statistics (Newey and West, 1987) are reported.
Sample consists of monthly alphas for active mutual funds and ETFs in the “Intermediate-Term Bond” Morningstar category, with Bloomberg Barclays US Aggregate Bond Index as their primary prospectus benchmarks and 10-year return histories.
REFERENCES


ii This table first appeared in Moore (2017).

iii Gerakos et al. (2016) state that the outperformance of their actively managed institutional accounts between 2000 and 2012 is explained almost entirely by tactical factor tilts.
This paper contains hypothetical analysis. Hypothetical and simulated examples have many inherent limitations and are generally prepared with the benefit of hindsight. There are frequently sharp differences between simulated results and the actual results. There are numerous factors related to the markets in general or the implementation of any specific investment strategy, which cannot be fully accounted for in the preparation of simulated results and all of which can adversely affect actual results. No guarantee is being made that the stated results will be achieved.

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