Investing in Retirement

“41 percent of Americans are at risk of running out of money in retirement.”


Financial planners, investment advisors and economists have often referred to the “three-legged stool” of retirement income: pension benefits, personal savings and Social Security benefits. Pension benefits are usually provided by an employer through a defined benefit (DB) plan. DB plans seek to provide a stable income for life, akin to an annuity. Personal savings include 401(k) balances and other savings in taxable accounts. Lastly, Social Security is an inflation-indexed monthly payment provided to the recipient for life by the government, where the amount of the benefit is based on an individual’s earning history.

As they reach retirement, individuals face a complex set of interrelated decisions for each of these three legs. They must decide:

1. Whether or not to defer Social Security benefits;
2. If they should buy an immediate or deferred annuity;
3. How to allocate their personal savings portfolio to various asset classes;
4. How much they plan to spend each year.

We have developed an optimization model to help answer these questions simultaneously. While most studies on investing in retirement rely on long-term return averages for asset returns, our capital markets assumptions reflect our views of future market conditions. For example, we assume that the 10-year rate will move from 2.5% to an average terminal value of 4% nominal in 40 years, which contrasts with historical studies that span long periods of declining interest rates. We also forecast inflation at 2.25% and an equity risk premium of 4% (2% in geometric terms) over the next 40 years.
We conclude that most retirees should consider allocating a relatively small portion of their savings (10%-20%) to a deferred annuity (where the purchaser exchanges an up-front premium for payments that start at some pre-determined point in the future), avoid immediate annuities and invest the remainder. Then we use sensitivity analysis on the whole system to show how one decision might influence the others. While these results are robust to a broad range of assumptions, our simultaneous optimization model also reveals important relationships between the various decisions retirees face. The inescapable conclusion of our analysis is that when investment managers recommend asset allocations and develop products for investing in retirement, such as income portfolios, they must ask “what is best for the retiree?” and seek to answer the question in a holistic fashion that accounts for all three legs of the retirement stool.

**Previous research**

In contrast, previous research has mostly focused on only one of these decisions at a time. Regarding the decision to annuitize personal savings, it has been well documented that immediate annuities are generally unappealing to investors due to the fact that they are comparatively expensive, lack liquidity and prevent retirees from passing on the investment as an inheritance (see, for example, Milevsky, 1998; Horneff, Maurer, Mitchell and Dus, 2006; Scott, Watson and Hu, 2006; Mitchell, Poterba, Warshawsky and Brown, 1999; and Sexauer, Peskin and Cassidy, 2012). In a recent editorial in which he reviews important papers on the topic, Siegel (2015) summarizes the issues with annuities as follows:

“Annuities! Everybody loves them on paper – they replace DB income streams and capture huge gains from pooling longevity risk – but nobody buys them. Even Jane Austen [“patron saint of annuitants”] would probably shun annuities today, given their inflexibility, high fees, adverse selection, and lack of transparent market.”

A common measure of annuity prices is the “money’s worth,” defined as the ratio of the present value of annuity benefits to the initial premium. A money’s-worth value of 100% would mean that the individual can expect to receive (in present value terms) annuity payments that exactly offset their initial premium. Such a mythical financial object is called an “actuarially fair” annuity. In practice, annuities are expensive: Estimates of money’s worth of initial premium range from 60% to 90% (Brown, Mitchell and Poterba, 1999; James and Song, 1999; Mitchell, Poterba, Warshawsky and Brown, 1999).

However, studies have shown the appeal of deferred annuities despite their high cost. Scott, Watson and Hu (2006) argue that deferred annuities are highly efficient and preferable to immediate annuities. Sexauer, Peskin and Cassidy (2012) suggest that the minimum risk portfolio for a retiree comprises 88% in a laddered Treasury Inflation-Protected Securities (TIPS) portfolio and 12% in a deferred annuity that starts when the TIPS payouts end. Similarly, Scott (2008) recommends an initial allocation of 10%-15% in a deferred annuity. In general, deferred annuities may be an effective tool to manage longevity tail risk. As Waring and Siegel (2014) put it, “Running out of money before running out of life is a catastrophe!” and a deferred annuity provides a hedge against this risk. Additionally, recent regulations allow for qualified deferred annuity purchases to satisfy minimum withdrawal requirements for tax-free retirement accounts, such as 401(k) and IRA plans, further increasing their appeal.

The annuitization of personal savings is only part of the challenge of investing in retirement. Other studies have focused on path-dependent decisions, such as adjusting spending rates and asset allocation over time. Horneff, Maurer, Mitchell and Dus (2006) introduce a flexible optimization model that allows a hypothetical retiree to choose nominal annuities, various investment allocations and various withdrawal strategies. They conclude that it’s preferable for a retiree to withdraw a fixed percentage of their savings every year than to buy immediate annuities. They also suggest that retirees should keep the option to switch into an annuity as needed. However, they do not evaluate deferred annuities. Similarly, in Fullmer’s (2007) model, the retiree switches to an annuity only if wealth falls to the level of the annuity premium, maximizing the value of the option.
Regarding the Social Security leg of the retirement stool, retirees can elect to receive their benefits anytime between the ages of 62 and 70. On average, Social Security benefits replace roughly 40% of pre-retirement income, though it can replace 80% or more for the very lowest income households and is the primary source of retirement income for the majority of retirees in the U.S. (Poterba, 2014). Scott (2012) argues that retirees should delay their Social Security benefits, as the net present value impact of doing so is positive: He describes the decision of when to claim Social Security as “perhaps the most important decision people will ever make.” However, as with most studies on investing in retirement, he evaluates this decision independently of the other decisions retirees face.

**Our model**

We have built a framework to optimize the mix of sources of retirement income across the three legs of the retirement stool. In contrast to the existing literature, we simultaneously account for as many decisions as possible. In our model, the investor determines how much of their initial wealth to devote to an immediate annuity, how much to devote to a deferred annuity, whether or not to defer collection of Social Security benefits and, for each year in retirement, how much they should spend. We also account for various asset allocations in the savings portfolio.

We assume that individuals have standard risk aversion preferences as follows (see, for example, Merton, 1969, and Samuelson, 1969):

$$U(c_t) = \frac{c_t^{1-\rho}}{1-\rho}$$  \hspace{1cm} (1)

Where $c_t$ represents the money the retiree spends (“consumption”) at year $t$, and $\rho$ is used to calibrate risk aversion. With these preferences, a higher value for $\rho$ means the retiree is more risk averse.

The retiree’s personal savings balance evolves through time according to the returns on stocks and bonds and the corresponding asset allocation, as follows:

$$w_{t+1} = (\varphi(1 + e_t) + (1 - \varphi)(1 + u_t))w_t s_t$$  \hspace{1cm} (2)

Where $w_t$ is wealth (the non-annuitized savings balance) at period $t$, $s_t$ is the fraction of wealth saved for the next period, $e_t$ and $u_t$ are the returns to stocks and nominal government bonds, respectively, and $\varphi$ is the allocation to stocks.

Real consumption in retirement is given by the sum of 1) Social Security payments, 2) any annuity payments and 3) the amount of current wealth spent rather than saved:

$$c_t = \begin{cases} SS_t + A + (1 - s_t)w_t & \text{for } t < 20 \\ SS_t + A + B + (1 - s_t)w_t & \text{for } t \geq 20 \end{cases}$$  \hspace{1cm} (3)

With $c_t$ equal to real consumption in period $t$, $SS_t$ is the Social Security payment, $A$ is the total immediate annuity payments, $B$ is the total deferred annuity payments (assuming the deferred annuity begins 20 years after retirement) and $(1 - s_t)w_t$ is the fraction of wealth spent in the current period. Note that the Social Security payments are indexed by $i$, which takes a value of zero or one (i.e., $\{0,1\}$). If the retiree chooses to start receiving benefits immediately, they receive a yearly payment of $SS_0$. If the retiree defers their Social Security benefits by five years, they receive a larger amount, which we denote as $SS_1$ (note that $SS_1$ equals zero for the first five years). For notational purposes, we refer to the fraction of initial wealth invested in the immediate annuity as $X_A$ and the fraction of initial wealth invested in the deferred annuity as $X_B$.

Altogether, the retiree maximizes the sum of expected discounted utility according to the following optimization, subject to the constraints expressed in equations (2) and (3):

$$\max_{\{s_t\}_{t=0}^T, X_A, X_B, \{SS_0, SS_1\}} \sum_{t=0}^T E_{qs_t u_t \pi_t} [\beta^t U(c_t)]$$  \hspace{1cm} (4)
Where $\pi_t$ is the mortality rate and $\beta$ is the discount rate that the individual applies to the value of future consumption.\(^3\)

Other assumptions and parameters include:

- In our baseline example, we assume the retiree is an unmarried 65-year-old male with a relative risk aversion of 4 and annual discounting of 2.5% ($\beta = 97.5\%$).\(^4\)
- The individual enters retirement with total wealth equal to five times his annual income.
- Social Security benefits cover 40% of pre-retirement income. This level is intended to represent the benefit for the median household (Klein and Pedersen, 2014, and Poterba, 2014).
- The annuities are priced given prevailing real interest rates for using a money’s worth value of 80%, which is slightly more generous than typical estimates in the literature (Mitchell, Poterba, Warshawsky and Brown, 1999).
- We initially consider a constant allocation in the savings portfolio of 40% stocks and 60% nominal government bonds, though we investigate the effect of other allocations.
- We assume that the wealth at retirement lies within a defined contribution plan, such as a 401(k) or IRA. The funds are accumulated pre-tax and, as such, annuity payments are fully taxable and are taxed at the same rate as Social Security payments. This is a simplifying assumption that we use for analytical convenience.\(^5\)

**Results**

With this set of assumptions, our retiree elects to receive Social Security immediately at age 65, spend 14% of their pre-retirement income on a deferred annuity (and nothing on an immediate annuity) and invest the remainder. Also, our framework solves for how much the retiree plans to spend each year. Figure 1 shows the optimal spending schedule across various capital markets scenarios, excluding spending...
from Social Security benefits. The dotted line shows the optimal spending plan as a percentage of the remaining savings balance, while the multiple paths represent absolute spending normalized by initial wealth — hence there is one absolute spending path for each capital markets path.

These results aren’t too far from the commonly used rule of thumb that retirees should spend 4% of their initial wealth each year during retirement (Bengen, 1994), although they allow for higher spending under most capital markets scenarios. Importantly, in our model, the percentage of savings to spend each year \((1 - s_t)\) is predetermined by the optimization, and the total amount spent \(((1 - s_t)w_t)\) varies as a function of the value of the remaining savings \((w_t)\), which itself depends on market returns. This mechanism optimally regulates spending: When wealth increases, the retiree plans on spending more; when wealth decreases, the retiree plans on spending less to ensure that wealth lasts as long as necessary. Despite some variability created by this rule, consumption is not extremely volatile. As Figure 1 shows, within the first 10 years, the 10th to 90th percentile range is between 4% and 7%. Over the following 10 years the range increases to 2.6% to 6.5%. Investors who wish to smooth consumption volatility can increase the risk aversion level in the model, or perhaps simply save more than planned following strong market returns.

Across scenarios, the optimal strategy is always to spend the entire balance of personal savings by the time the deferred annuity payments start. As the dark line in Figure 1 shows, optimal spending as a percentage of remaining savings increases sharply over time as the savings balance gets smaller and smaller. In this example, the personal savings account is “cleaned up” (with 100% of remaining wealth spent) in the last year before the retiree reaches 85 years old, across all capital markets scenarios. In theory, savings are never completely depleted before the deferred annuity begins, since spending is defined as a percentage of the remaining balance. However, the balance may tend toward zero under extremely bad market conditions. In this unlikely — but theoretically possible — scenario, the retiree would eventually subsist solely on their Social Security benefits until the start of the deferred annuity payments, but retirement consumption would be decreased smoothly and gradually rather than fall abruptly once a retiree’s savings are fully depleted. After 85, total spending remains stable across all capital markets scenarios, reflecting the deferred annuity rate of 4.8%. The line is flat because we assume a real annuity. All numbers are reported after adjusting for inflation.

In sum, the deferred annuity provides a longevity safety net. Without it, the retiree must substantially delay spending in order to make the money last as long as possible, as shown in Figure 2. This delay results in lower overall real consumption, particularly later in retirement.

**FIGURE 2: PERCENT OF REMAINING SAVINGS SPENT – WITH AND WITHOUT DEFERRED ANNUITY**

![Figure 2: Percent of Remaining Savings Spent – With and Without Deferred Annuity](image)

<table>
<thead>
<tr>
<th>Percentage of remaining wealth spent each period</th>
<th>0%</th>
<th>20%</th>
<th>40%</th>
<th>60%</th>
<th>80%</th>
<th>100%</th>
</tr>
</thead>
<tbody>
<tr>
<td>Age</td>
<td>65</td>
<td>68</td>
<td>71</td>
<td>74</td>
<td>77</td>
<td>80</td>
</tr>
</tbody>
</table>

Note: Sensitivity analysis on the optimal allocation to annuities
Source: PIMCO simulation results. Refer to appendix for additional detail.

Our results are stable across a wide range of assumptions. Figure 3 shows the results of a sensitivity analysis on the optimal allocation to annuities without allowing for any delay of Social Security benefits. For this analysis, we move one parameter at a time and leave the baseline set of assumptions...
unchanged for all other variables. The optimal fraction of pre-retirement wealth allocated to annuities ranges from 8% to 20%: According to this analysis, immediate annuities are never worth the cost over a range of realistic prices, parameters and circumstances.

**FIGURE 3: SENSITIVITY ANALYSIS ON THE PERCENTAGE ALLOCATED TO ANNUITIES WITHOUT SOCIAL SECURITY DELAY**

<table>
<thead>
<tr>
<th>Gender and marital status</th>
<th>% Deferred</th>
<th>% Immediate</th>
</tr>
</thead>
<tbody>
<tr>
<td>Male</td>
<td>14.3%</td>
<td>0%</td>
</tr>
<tr>
<td>Female</td>
<td>19.3%</td>
<td>0%</td>
</tr>
<tr>
<td>Married</td>
<td>20.9%</td>
<td>0%</td>
</tr>
<tr>
<td>Female</td>
<td>14.1%</td>
<td>14.3%</td>
</tr>
<tr>
<td>Married</td>
<td>14.5%</td>
<td>0%</td>
</tr>
<tr>
<td>Male</td>
<td>0%</td>
<td>16.0%</td>
</tr>
<tr>
<td>Female</td>
<td>0%</td>
<td>14.3%</td>
</tr>
<tr>
<td>Married</td>
<td>0%</td>
<td>12.5%</td>
</tr>
<tr>
<td>Male</td>
<td>0%</td>
<td>0%</td>
</tr>
<tr>
<td>Female</td>
<td>0%</td>
<td>0%</td>
</tr>
<tr>
<td>Married</td>
<td>0%</td>
<td>0%</td>
</tr>
</tbody>
</table>

% Deferred | % Immediate
---|---
20% (Base case) | 60%
5%
2.5% (Base case) | 16.0%
14.0%
0%
14.3%
14.6%
0%
13.1%
14.3%
14.6%
0%
14.5%
14.3%
13.8%
0%
14.8%
14.3%
13.8%
0%

Source: PIMCO simulation results. Refer to appendix for additional detail.

This analysis also reveals the following findings, most of which are self-evident:

- **Life expectancy matters.** Females have higher life expectancy, and therefore should invest a larger amount in the deferred annuity. Married couples should invest even more, because the probability that one of them lives longer than expected is higher than either of the individual probabilities.

- **The choice of asset allocation in the savings portfolio seems to have little impact, although a higher allocation to stocks leads to a slightly larger amount invested in the deferred annuity. As the riskiness of the investment portfolio increases, the volatility (and mean) of retirement consumption increases. To hedge part of this risk, our retiree sets aside additional wealth in the deferred annuity.**

- **As the discount rate increases, individuals value present consumption more than future consumption. Therefore, the higher the discount rate, the lower the allocation to the deferred annuity, which allows for additional spending earlier in retirement.**

- **Individuals who are more risk-averse should invest a larger amount in the deferred annuity to protect for the risk of not having enough income should they live beyond their life expectancy. Although, in general, risk aversion seems to have a modest effect overall.**

- **Regarding income replacement rates, as Social Security replacement rates fall, individuals should invest a higher percentage in the deferred annuity, while individuals with larger pensions already have a fair amount of income-for-life security and therefore can afford a smaller allocation. The 25% replacement rate reflects a cut in benefit replacement rates as might be the case for high-income households or in the event of cost-cutting Social Security reforms. The 73% income replacement from pension payments is obtained by adding a private pension payment of $2,100 per month, which reflects the average state and local government monthly pension payment in 2011, according to the U.S. Census.**

- **The price of the annuity has a surprisingly small effect on the optimal annuity purchases. As the money’s worth declines and the annuity becomes more expensive, total spending on deferred annuities increases slightly. This result may appear counterintuitive but can be explained by the fact that if money’s worth declines, retirees must spend more to receive an annuity that generates the same amount of income.**

**Social Security**

When we remove the restriction on deferring Social Security benefits in exchange for higher payments, we find that the decision whether to delay Social Security interacts with annuity purchases in interesting ways. Figure 4 shows the optimal immediate and deferred annuitization rates as life expectancy varies, with and without delaying Social Security. When Social Security is delayed, the higher payments provide a substitute
for the longevity safety net that was provided by the deferred annuity, and the optimal allocation to the deferred annuity declines substantially for the highest life-expectancy retirees due to this effect.

FIGURE 4: EFFECT OF DELAYED SOCIAL SECURITY BENEFITS ON ANNUITIZATION

<table>
<thead>
<tr>
<th></th>
<th>% Deferred</th>
<th>% Immediate</th>
<th>Delay optimal?</th>
</tr>
</thead>
<tbody>
<tr>
<td>Male (Base case)</td>
<td>No delay</td>
<td>14.3% 8.6%</td>
<td>0.0% 0.0%</td>
</tr>
<tr>
<td></td>
<td>Delayed SS</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Female</td>
<td>No delay</td>
<td>19.3% 8.5%</td>
<td>0.0% 0.0%</td>
</tr>
<tr>
<td></td>
<td>Delayed SS</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Married</td>
<td>No delay</td>
<td>20.9% 8.6%</td>
<td>0.0% 0.0%</td>
</tr>
<tr>
<td></td>
<td>Delayed SS</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Male - No annuities</td>
<td>No delay</td>
<td>0.0% 0.0%</td>
<td>0.0% 0.0%</td>
</tr>
<tr>
<td></td>
<td>Delayed SS</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Source: PIMCO simulation results. Refer to appendix for additional detail.

Note that in our baseline case, we find that the retiree should not delay Social Security benefits; this recommendation is contrary to Scott’s (2012) conclusion, where he demonstrates that delaying Social Security provides a substantial gain in terms of net present value of consumption. We agree with Scott’s (and many others’) calculation of the positive net present value of delaying Social Security. However, in our model, we account for a more complete set of interrelated decisions, and with our base-case set assumptions, we conclude that Social Security receipt substantially lowers the volatility of early retirement consumption and provides an attractive safety net against the risk of ruin until deferred annuity payments begin. Also, note that for females or married couples, delaying Social Security becomes attractive, as the longer period of higher benefit receipt associated with delaying Social Security payments outweighs the increase in risk earlier in retirement.6

Conclusions

With the continued decline of defined benefit pension plans, retirees must find ways to replace this leg of their personal retirement stools. They must translate accumulated wealth at retirement into income throughout retirement. This is no easy feat.

As they reach retirement age, retirees face a complex set of interrelated investment decisions. While a great deal of effort has been made to try to optimize these decisions, most of the existing work evaluates each option independently, or “holding everything else constant.” Instead, we have built a model to optimize the most significant retirement decisions simultaneously, and analyze the effect of each decision and each variable on the entire system. We have shown that with this integrated perspective, delaying Social Security benefits or increasing fixed income investments may be an appealing substitute for deferred annuities.

Looking for broad recommendations on how to invest in retirement is like looking for a shoe size that will fit everyone. Every individual’s situation is different, and solutions that focus on the “typical” retiree are unlikely to fit many individuals. Nonetheless, we find that for a wide range of situations, allocating 10%–20% of retirement wealth into a deferred annuity and investing the remainder appears optimal. We reach this conclusion even without consideration of some of the most commonly cited advantages of self-directed investment, such as flexibility to change spending over time or to handle unanticipated expense.

Our framework is broad and flexible enough to open the door to several possible extensions. For example:

- Developing dynamic asset allocation strategies in the remaining savings balance;
- Linking pre-retirement accumulation strategies with in-retirement investing;
- Modifying the utility function to more directly account for habit formation and extreme loss aversion below a (poverty) threshold;
- Developing dynamic tax-aware strategies;
- Directly accounting for housing wealth;
- Adding bequest motives to the model,
- Modeling with credit risk in annuities, and
- Accounting for large unexpected expenses, such as sudden healthcare costs.

Many of these questions can be addressed indirectly by calibrating parameters in the model (such as risk aversion), while others may require additional variables or modest extensions to the model. An important takeaway is that our framework provides a strong foundation upon which many degrees of complexity can be added.

Overall, retirees have many options available to transfer their savings into income, and investing in retirement is one of the most complex problems in investment research. After all, as Siegel (2015) puts it:

“(…) Despite thousands of scholarly and practical articles and much earnest effort by researchers, financial product designers, pension plan sponsors, advisers, legislators, regulators, and individual investors, we still have a retirement crisis.”

In this context, we do not claim to have built the definitive model to solve the retirement crisis, but we hope our effort will contribute to the discussion.

References


Appendix: Simulation Methodology and Assumptions

Realized inflation is tied to the dynamics of break-even inflation rates implied by the joint simulation of real and nominal rates. Both real (inflation-linked) and nominal bond returns at all relevant maturities are directly determined by the simulated yield curves. When real rates are low, real income from a given mix of assets (especially fixed income) is low.

Nominal rates process

A reduced form three-factor log-normal Cox-Ingersoll-Ross (CIR) model governs the dynamics for the term structure of nominal rates as follows:

\[ dy_2 = (\Theta_{2,y} - y_2)dt + \sigma_{2,y}\sqrt{y_2}dz_{2,y} \]  
\[ dy_{10} = (\Theta_{10,y} - y_{10})dt + \sigma_{10,y}\sqrt{y_{10}}dz_{10,y} \]  
\[ dy_{30} = (\Theta_{30,y} - y_{30})dt + \sigma_{30,y}\sqrt{y_{30}}dz_{30,y} \]

Where \( y_t \) is the level of nominal yields of maturity \( t \), \( \Theta_{t,y} \) is the terminal, or long-run, level of nominal yields of maturity \( t \), \( \sigma_{t,y} \) is the volatility of nominal yields of maturity \( t \) and \( z_{t,y} \) is the shocks to nominal yields of maturity \( t \) (which are correlated across maturities).

Real yields

A reduced form three-factor Vasicek model governs the dynamics for the term structure of real rates, as follows:

\[ dr_2 = (\Theta_{2,r} - r_2)dt + \sigma_{2,r}r_2dz_{2,r} \]  
\[ dr_{10} = (\Theta_{10,r} - r_{10})dt + \sigma_{10,r}r_{10}dz_{10,r} \]  
\[ dr_{30} = (\Theta_{30,r} - r_{30})dt + \sigma_{30,r}r_{30}dz_{30,r} \]

Where \( r_t \) is the level of real yields of maturity \( t \), \( \Theta_{t,r} \) is the terminal level of real yields of maturity \( t \), \( \sigma_{t,r} \) is the volatility of real yields of maturity \( t \) and \( z_{t,r} \) is the shocks to real yields of maturity \( t \) (which are correlated across maturities).

The term structures for both nominal and real rates are fitted using a four-factor Nelson-Siegel model.

Inflation and breakeven inflation

The process for the term structure of break-even inflation is defined by the difference between nominal and real rates. For example, we define the short-term break-even inflation as the difference between the simulated one-year nominal rate and the simulated one-year real rate, as follows:

\[ \pi_{1,t} = y_{1,t} - r_{1,t} \]  
\[ \pi_{t,t} \] represents break-even inflation rates and \( r_{1,t} \) and \( y_{1,t} \) are defined as before.

Realized inflation in a given year is then simulated as the break-even inflation plus a stochastic term

\[ \pi_{\text{infl}} = \pi_{1} + \sigma_{\text{infl}}\nu_{\text{infl}} \]

Where \( \sigma_{\text{infl}} \) being realized inflation volatility and \( \nu_{\text{infl}} \) being the shocks to inflation at time \( t \).

In this way, the actual realized inflation process is consistent with the short-term break-even inflation rate at each point in the simulation. Consumption throughout retirement is inflation-adjusted by the cumulative realized inflation rate along each path.
Equity returns

Simulated real equity returns are consistent with the simulated interest rate processes, and then based on the following specification:

\[ x_{eq} = r_1 + \lambda_{eq} + \sigma_{eq} d z_{eq,t} \]  \hspace{1cm} (13)

Where \( r_1 \) is the level of one-year real yields, \( \lambda_{eq} \) is the equity risk premium, \( \sigma_{eq} \) is the volatility of equities and \( z_{eq,t} \) is the shocks to equity returns at time \( t \).

Parameter assumptions

Figure 5 summarizes our parameter assumptions for the simulation of asset returns, yield curves and inflation.

**FIGURE 5: PARAMETER ASSUMPTIONS**

<table>
<thead>
<tr>
<th>Variable</th>
<th>Parameter</th>
<th>Value</th>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Nominal rates</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2-year nominal key rate</td>
<td>( \theta_{2,y} )</td>
<td>3.50%</td>
<td>( \sigma_{2,y} )</td>
<td>150 bps vol.</td>
</tr>
<tr>
<td>10-year nominal key rate</td>
<td>( \theta_{10,y} )</td>
<td>4.00%</td>
<td>( \sigma_{10,y} )</td>
<td>100 bps vol.</td>
</tr>
<tr>
<td>30-year nominal key rate</td>
<td>( \theta_{30,y} )</td>
<td>4.25%</td>
<td>( \sigma_{30,y} )</td>
<td>85 bps vol.</td>
</tr>
<tr>
<td>Real rates</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2-year real key rate</td>
<td>( \theta_{2,r} )</td>
<td>1.00%</td>
<td>( \sigma_{2,r} )</td>
<td>100 bps vol.</td>
</tr>
<tr>
<td>10-year real key rate</td>
<td>( \theta_{10,r} )</td>
<td>1.25%</td>
<td>( \sigma_{10,r} )</td>
<td>85 bps vol.</td>
</tr>
<tr>
<td>30-year real key rate</td>
<td>( \theta_{30,r} )</td>
<td>1.50%</td>
<td>( \sigma_{30,r} )</td>
<td>65 bps vol.</td>
</tr>
<tr>
<td>Inflation</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Realized inflation</td>
<td>( E[\pi_1] )</td>
<td>2.25%</td>
<td>( \sigma_{infl} )</td>
<td>60 bps</td>
</tr>
<tr>
<td>Equity</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Equity risk premium (arithmetic)</td>
<td>( \lambda_{eq} )</td>
<td>4.0%</td>
<td>( \sigma_{eq} )</td>
<td>20% vol.</td>
</tr>
<tr>
<td>Equity risk premium (geometric)</td>
<td>( \lambda_{eq} - 0.5 \sigma_{eq}^2 )</td>
<td>2.0%</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Overall, parameters are calibrated to produce realistic long-term dynamics for bond returns, equity returns, nominal and real rates and inflation. Note that these long-run values are for a 40-year horizon and they are broadly in line with Federal Reserve projections. The volatility levels are calibrated to be in line with historical experience. We assume a long-term average inflation rate of 2.25%, and we build in a term premium in the yield curve of 50 basis points (bps) at the 10-year maturity and 75 bps for 30-year rates. The real yields converge to 1%, 1.25% and 1.5% for the 2-year, 10-year and 30-year maturities. The initial values for the 2-, 10- and 30-year real yields are –1.5%, 1% and 1.5%, while the initial nominal yields are 0.5%, 2.5% and 4% (again for the 2-year, 10-year and 30-year points). Consistent with empirical data, real yields are assumed to be somewhat less volatile than nominal yields in our simulation.

The equity risk premium is assumed to be 4% in arithmetic terms and 2% in geometric terms, which is considerably lower than the long-term historical averages of 7.5% and 5.5%, respectively. The moderate equity risk premium is consistent with current pricing in equity markets and also consistent with a gradual reduction in the ex-ante equity risk premium that investors have required in global equity markets.
Twenty years roughly corresponds to life expectancy. Also, for tax reasons, a deferred annuity is considered a qualified withdrawal only if it is deferred 20 years or less, which is another reason why it is common industry practice to use 20-year deferred annuities.

In practice, individuals may defer Social Security for any length of time up to their 70th birthday and gradually increase their benefits; we use no delay or a delay of the full five years for computational efficiency.

Mortality rates are calibrated to match gender-specific survival probabilities for 65-year-olds from the Social Security Administration.

Note that this discount rate is not required to equal the inverse interest rate, as there is no inter-temporal market clearing condition in the model; we assume that our retiree is a price-taker whose actions do not influence financial prices.

Every individual’s tax position is different based on income bracket, state of residence, realized capital gains and so on. Our model assumes constant and equal taxation rates across different income sources.

In practice, the retiree may activate Social Security along the way if markets crash and the savings balance becomes insufficient. However, as mentioned, for computational efficiency we did not model this path-dependent decision. Also, since Social Security is not fully funded, there is a risk that promised benefits get reduced. Lower-than-expected Social Security benefits would likely increase the need for annuities, holding risk aversion and other parameters constant, as we show in Figure 3.
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Statements concerning financial market trends or portfolio strategies are based on current market conditions, which will fluctuate. There is no guarantee that these investment strategies will work under all market conditions or are suitable for all investors and each investor should evaluate their ability to invest for the long term, especially during periods of downturn in the market. Investors should consult their investment professional prior to making an investment decision.

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