PIMCO

The Value of Smoothing

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Executive Summary

- Although most research on private assets asks whether returns are commensurate with risk, we focus on the value of the smoothed volatility profile of private assets that are not marked to market.
- The smoothing of returns in private equity has a substantial impact on headline volatility. We find that the true economic volatility of private equity is close to 30%, versus a headline number of 10%.
- Based on reasonable assumptions, we find that smoothing results in an almost 0% probability of observing a 30% drawdown, versus a true probability of 15%–16% over a three-year period.
- Under the same parameters, the expected observed maximum drawdown is 12% under smoothing, versus a true value of 40%.
- To be indifferent about the choice between a smoothed private index and a public index with similar risk, a representative investor based on our analysis would require the public index to have 6 percentage points of additional return annually.
- Smoothing also protects investors from their behavioral demons, such as the tendency to buy high and sell low. For a 10-year horizon, we find a reasonable estimate of the gain from locking up money in a private equity "straitjacket" is an extra annual return of 1.7%.

INTRODUCTION

In the face of strained valuations and low yields, private investments have moved front and center for investors as a pocket of the market that continues to target high single-digit and double-digit returns. The demand for returns, combined with the opacity and difficulty of getting data on private funds, has spawned a huge debate in the literature on private asset performance.[1] In this piece, we choose to stay away from the "food fight" on whether private managers can or cannot generate alpha.[2] Instead, we focus on the value of the smoothed volatility profile of select private assets, which self-appraise their investments and are not required to mark to market. For example, private equity buyouts have an observed return volatility of 10%, but their true economic return volatility should be closer to 30%.

1 Harris et al. (2014) find private equity consistently outperforms public markets, while Phalippou (2020) reports that post-fee private equity performance is about the same as that of public equity indices. See Robert S. Harris, Tim Jenkinson and Steven N. Kaplan, "Private Equity Performance: What do we know?" *Journal of Finance*, October 2014, and Ludovic Phalippou, "An Inconvenient Fact: Private Equity Returns & The Billionaire Factory," working paper, 2020.

2 Michael Cembalest, "Food Fight: An update on private equity performance vs public equity markets," J.P. Morgan Asset and Wealth Management, 2021.

This substantial gap in volatility is an illusion driven by the lack of trading and different regulations around the valuation of private assets. This charade may only be revealed with a large market correction in which prices do not recover within the life of the fund. Given the remarkable bull cycle and fast recoveries from drawdowns over the past two decades, there has not been such an episode. In this paper, we use a few simple thought experiments to quantify how much the illusion of low volatility in private returns may be worth in terms of job security, wealth, disaster risk and the reduction of behavioral biases.

These benefits are not "real," because the final portfolio performance of private assets will likely be dictated by the true risk underlying these investments. Smoothing does not add returns to the portfolio. Our results illustrate the hypothetical value of smoothing for an investor in units that are easy to interpret. In what follows, we use private equity as an example, taking the more conservative view that buyouts have a true economic volatility of 24%, more in line with the Russell 2000 Value Index. We find that smoothing results in almost no chance of observing a 30% drawdown, versus a true probability of 15%–16% over a three-year period, and the expected maximum drawdown is 12%, compared with a true value of 40% over the same horizon. To be indifferent about the choice between a smoothed private index and a public index of similar risk, a representative investor based on our analysis would require 6 percentage points of additional return annually on the publicly traded index. Smoothing also protects investors from their behavioral demons, such as the tendency to buy high and sell low. For a 10-year horizon, we find the private equity "straitjacket" gives investors an estimated 1.7% extra return annually.

A QUICK LOOK AT THE DATA

We begin with an illustration of the degree of smoothing in private assets compared with public benchmarks. In Exhibit 1, we use the aggregate benchmark indices from Preqin to measure private asset performance.

Exhibit 1: Private versus public indices

12/2000 - 3/2021	Index	Return	Standard deviation	Sharpe	Max drawdown
Preqin indices	Private capital	9.62%	8.32%	0.94	-26.69%
	Buyouts	12.41%	9.86%	1.08	-28.78%
	Real estate	8.15%	9.35%	0.68	-47.69%
	Private debt*	6.96%	8.27%	0.72	-26.30%
Public indices	S&P 500	7.84%	17.08%	0.35	-46.53%
	Russell 2000 Value	9.87%	23.73%	0.37	-43.70%
	MSCI US REIT Index	9.77%	21.91%	0.36	-66.46%
	Bloomberg High Yield Index (Duration Hedged)*	5.28%	13.82%	0.31	-32.78%

Source: Preqin, Bloomberg and PIMCO

* Private debt and the Bloomberg High Yield Index (Duration Hedged) are measured from December 2007 to March 2021, the latest data available. The Sharpe ratio is defined as the return in excess of three-month Libor divided by the volatility.

The observed volatilities of private assets are substantially lower than those of publicly traded indices. Clearly, it cannot be that private assets are much safer investments. This would be akin to saying leveraged buyouts are half as risky as buying the S&P 500!

We believe a lack of reporting requirements for private funds drives the low standard deviation observed in the Preqin indices. Private funds are not required to mark to market and instead use self-appraised values for their investments. Data vendors such as Cambridge Associates and Preqin rely on these net asset values in the construction of aggregate performance indices. In the literature, methodologies have been proposed to recover the true risk by "unsmoothing" these returns.[3] For illustrative purposes, we follow Kazemi et al. (2016) and assume the reported returns are a moving average of the lagged reported return and the true return.

3 Niels Pedersen, "Asset Allocation: Risk Models for Alternative Investments," PIMCO *Research*, 2013, and Hossein Kazemi, Keith Black and Donald Chambers, "Alternative Investments: CAIA Level II," Chapter 15, 2016.

Exhibit 2: Public versus private returns (moving average)

12/2000 - 3/2021	Index	Return	Standard deviation	Sharpe	Max drawdown
Preqin indices	Private capital	9.62%	8.32%	0.94	-26.69%
	Buyouts	12.41%	9.86%	1.08	-28.78%
	Real estate	8.15%	9.35%	0.68	-47.69%
	Private debt*	6.96%	8.27%	0.72	-26.30%
Unsmoothed	Private capital	9.76%	14.88%	0.54	-40.38%
	Buyouts	12.47%	16.10%	0.66	-38.81%
	Real estate	6.82%	17.29%	0.29	-63.10%
	Private debt*	6.77%	13.76%	0.42	-35.69%

Source: Preqin, Bloomberg and PIMCO. **For illustrative purposes only.**

* Private debt is measured from December 2007 to March 2021, the latest data available. The Sharpe ratio is defined as the return in excess of three month Libor divided by the volatility.

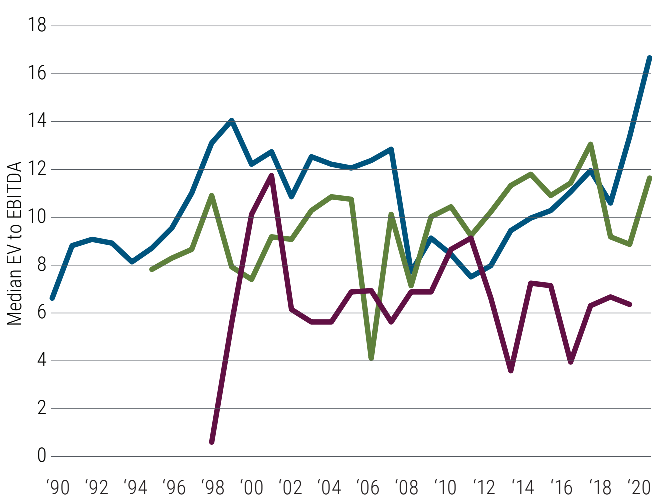
Unsmoothing does give us a meaningful boost in the standard deviations of returns. However, the procedure is highly sensitive to the number of lags and the assumed law of motion for the reported returns. To get a more reliable gauge of the true economic risk underlying private funds, we must look at the individual deals and cash flows underlying private funds. In the next section, we take a deeper look at buyouts.

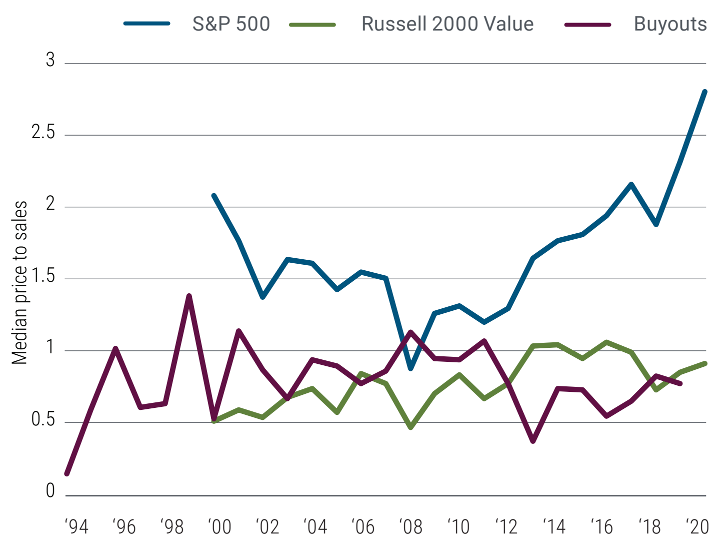
UNDER THE HOOD: PRIVATE EQUITY DEALS

Using Preqin data on North American buyouts, we find the median private equity deal is $56 million, significantly smaller than the market cap of any of the companies that trade in the S&P 500. Looking next at valuation multiples, we compare median enterprise value (EV)-to-EBITDA and the median price-to-sales ratio within each deal year with public indices (see Exhibit 3).

Exhibit 3: Comparing private buyout deals to public companies

	S&P 500	Russell 2000 Value	Buyouts
Average EV/ EBITDA	10.67	9.67	6.57
Average price to sales	1.68	0.78	0.79

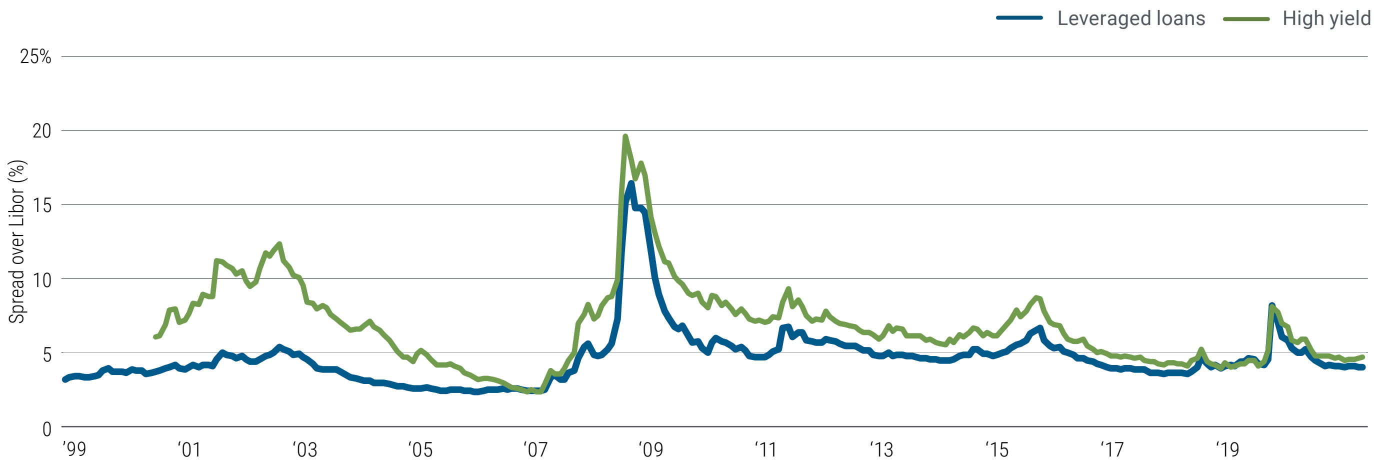




Source: PIMCO, Bloomberg and Preqin as of December 2021

Private equity managers tend to target "cheap" companies when viewed from the metrics of EV-to-EBITDA and price-to-sales. Based on size and valuation metrics, buyout deals tend to be more in line with a small cap value index like the Russell 2000 Value Index than with the S&P 500.[4] Lastly, we look at the capital structure. Private equity funds generally borrow to finance a buyout, typically using 33% equity and 66% debt,[5] while firms in the Russell 2000 Index have, on average, 70% equity and 30% debt. Another way to see the leverage of private equity deals is to analyze leveraged loans, which are the debt side of buyout transactions. In Exhibit 4, we compare the spread of leveraged loans with high yield spreads.

Exhibit 4: Comparing spreads on leveraged loans to high yield



Source: Bloomberg and S&P Global as of December 2021. Leveraged loans are represented by the S&P/LSTA Leveraged Loan Index. High yield bonds are represented by the Bloomberg High Yield Index (Duration Hedged).

With the exception of the pre-2008 period, when leveraged loans were a relatively new asset class, leveraged loans and high yield have traded at similar spreads, suggesting they have similar magnitudes of risk. If private equity and leveraged loans are the equity and debt pieces of the capital stack for buyout deals, we could look at the equity of firms with debt trading in the high yield category. In Exhibit 5, we compare the average leverage and equity return volatility for an index of firms trading in the high yield CDX index with public indices.

Exhibit 5: Leverage and volatility in equity of high yield firms versus public indices

	HY firms	S&P 500	Russell 2000 Value
Volatility of returns	25.3%	12.4%	16.9%
Average net debt to EV	50.2%	25.7%	45.6%

Source: Bloomberg and PIMCO. High yield firms refers to firms in the on-the-run high yield CDX index that have publicly traded equities. We then take an equal weight of these returns and compute the standard deviation of returns over time. Data covers period from January 2004 to December 2021.

4 We acknowledge the median market cap for companies in the Russell 2000 Value Index is $1.1 billion, which is still significantly higher than buyout deals. However, the Russell index is a widely used public index that captures the size and value components of private equity deals, so we choose it as a reference point.

5 Phalippou (2020)

The equity return is much more volatile for high yield firms than for firms in public indices. Thus, to construct a better public benchmark for private equity, we need to incorporate small, cheap, leveraged and volatile public equities.

Starting with all the U.S. firms within the MSCI universe of public stocks, we construct a custom index that filters within industries by size, cheapness, leverage and return volatility.[6] Exhibit 6 shows statistics for the resulting custom benchmark of publicly traded equities that we believe more closely match the investment style of buyouts.

Exhibit 6: Return statistics of our custom benchmark compared with public and private indices

12/2000 - 3/2021	Index	Return	Standard deviation	Sharpe	Max drawdown
Preqin indices	Private capital	9.62%	8.32%	0.94	-26.69%
	Buyouts	12.41%	9.86%	1.08	-28.78%
	Real estate	8.15%	9.35%	0.68	-47.69%
	Private debt	6.96%	8.27%	0.72	-26.30%
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	Buyouts	12.47%	16.10%	0.66	-38.81%
	Real estate	6.82%	17.29%	0.29	-63.10%
	Private debt	6.77%	13.76%	0.42	-35.69%
Custom index	Small, cheap, leveraged public equities	10.54%	29.81%	0.29	-58.80%

	HY firms	Custom benchmark
Volatility of returns	29.6%	26.4%

Source: PIMCO, Preqin and Bloomberg as of December 2021. **For illustrative purposes only.**

We find that when we match the investment style of buyout deals more closely, we should expect a true economic volatility closer to 30% than the 9.86% observed within the Preqin aggregate index – a very significant difference. Even behavioral effects that cause public assets to have excess volatility simply by being available to trade does not fully explain the large differential between the observed and true volatilities.[7]

To compare the performance of a private equity fund against our publicly traded custom index, we use the Kaplan-Schoar public market equivalent (PME).[8] The PME calculates the net present value of the cash flows of funds in the Preqin database, including calls and distributions, and uses the return of the custom index as the discount rate. The alpha[9] of the fund is the return that needs to be added to the return of the custom index in the discount rate such that the fund has a net present value of cash flows equal to zero. In this way, the alpha captures the additional return the fund provides compared with what would have been achieved had the capital calls and distributions been invested in the custom index. Exhibit 7 shows the distribution of alpha across funds by vintage.

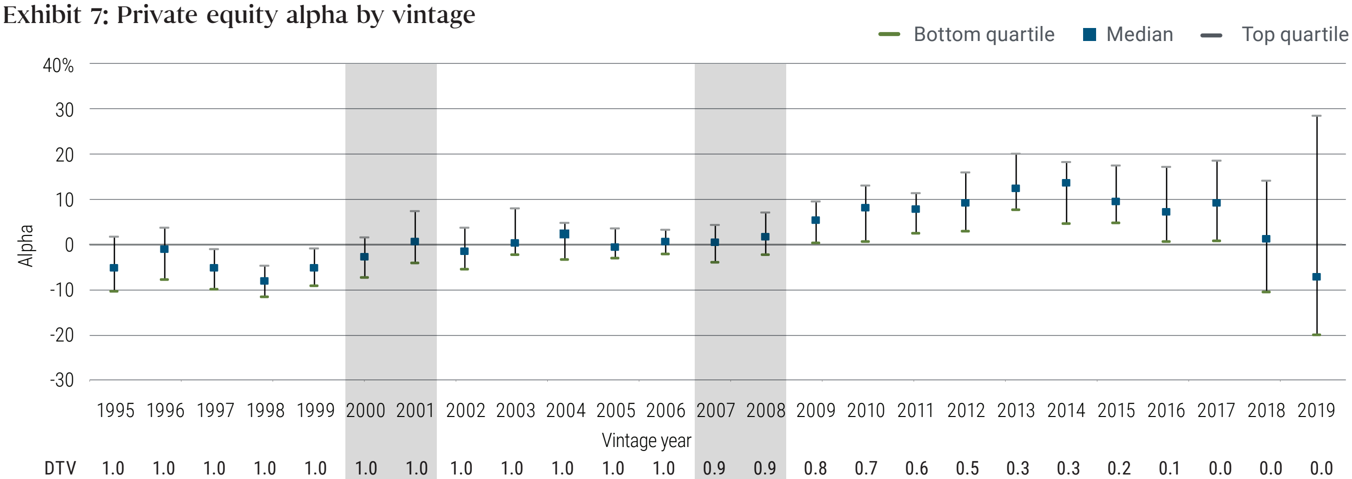
6 See appendix for more details.

7 Behavioral effects such as the weekend effect show the potentially excess volatility in publicly traded assets. See Kenneth R. French, "Stock returns and the weekend effect," *Journal of Financial Economics,* March 1980.

8 Steven N. Kaplan and Antoinette Schoar, "Private Equity Performance: Returns, Persistence, and Capital Flows," *Journal of Finance,* August 2005.

9 In this context, we refer to alpha as a combination of the illiquidity premium and the alpha generated by good deal selection on the part of the manager. From the PME calculation, it is impossible to disentangle the two without further assumptions.

Exhibit 7: Private equity alpha by vintage



Source: PIMCO and Preqin. **Hypothetical example for illustrative purposes only.** The exhibit is provided for illustrative purposes and is not indicative of the past or future performance of any PIMCO product.

We also include the distributions to total value (DTV), or the ratio of distributed capital to the total value of remaining investments plus the distributed capital. Interestingly, we find that for older vintages that have paid out most of their investments, the median fund based on vintage year has an average alpha close to zero. Recent vintages, however, which have not yet fully distributed their investments and rely on self-appraised values, have high positive alpha. As noted, we stay away from commenting too much on whether private equity outperforms or underperforms public equities but show the results using our custom index for completeness.

THE CASE FOR SMOOTHED RETURNS

Are investors deluding themselves by using artificially smoothed returns when evaluating private investments? The answer may not be so simple. Even if the true economic risk underlying private investments is not captured by appraisal values, the smoothed return series may be more appropriate for investors, for a variety of reasons.

First, private fund investors are often buy-and-hold, meaning short-term gyrations in markets may not affect their investment decision, and they may prefer smoothed returns that remove day-to-day volatility. Second, private deals can be very idiosyncratic, and finding an appropriate valuation may be difficult and resource-intensive; appraisal values remain a quick and easy way to evaluate performance. Third, the true performance of a deal is known only after the general partner (GP) finishes distributing capital. At this point, the smoothed series must eventually converge to the actual value. Thus, over a long enough horizon, the smoothed benchmark should give approximately the correct return for private funds, although the volatility may not be accurate.

In the following sections, we quantify the hypothetical value offered through the rose-colored lens of private fund accounting, using a few thought experiments. For illustrative purposes, we use the Preqin buyouts index and the Russell 2000 Value Index as private and public counterparts. The results below would be even more dramatic if we used our custom benchmark as our public proxy. Again, we emphasize that these are not "real" benefits to the investor but our attempt to convert the value of the illusion of smoothing into interpretable units.

PROBABILITY OF GETTING FIRED

Imagine you are the CIO of a pension fund. Your board sets you loose with the condition that if you perform significantly worse than your peers, you will be fired.

This scenario may not be too far-fetched – peer comparisons are commonly used to benchmark manager performance. In this scenario, we can calculate how much extra job security a "smoothed" private asset delivers. Assuming private fund returns follow a geometric Brownian motion, we can calculate the probability that the process hits a lower threshold within a certain time period (see appendix for details).

Applying this to the Preqin Private Capital Quarterly Index (restricted to North American buyout funds), we calculate the expected log return to be 12% and the smoothed volatility to be 10%. We compare this with an asset that has the same expected log return but a volatility of 24% (more in line with the Russell 2000 Value Index). Exhibit 8 shows the probability of reaching different thresholds over a three-year period.

Exhibit 8: Probability of a drawdown for different levels of volatility

3 years		Volatility	
Log return	**Return**	**10%**	**24%**
-11%	-10.0%	7.8%	60.2%
-22%	-20.0%	0.4%	32.9%
-36%	-30.0%	0.0%	15.6%
-51%	-40.0%	0.0%	6.1%

Source: PIMCO. Data covers period from December 2000 to December 2021.
For illustrative purposes only.

Over the three-year period, there is little chance of hitting a 30% loss when the asset has the smoothed volatility of the private index. However, if the asset behaves more like a publicly traded investment, there is a 15.6% chance it will be down 30%.

Another way to quantify this benefit is the expected max drawdown (see appendix for details). Using a 12% expected return, Exhibit 9 compares the expected max drawdown for the two levels of volatility over a three-year period.[10]

Exhibit 9: Expected maximum drawdown for different levels of volatility (log return)

3 years	Volatility	
	10%	**24%**
Expected value	12.4%	39.9%

Source: PIMCO as of December 2021. **For illustrative purposes only.**

Over the three-year period, the expected drawdown is almost 28% less for the smoothed private asset, meaning the expected worst loss would be 28% worse than peers using public rather than private assets – a remarkable incentive to rely on smoothed quarterly performance updates even if the accounting of private assets is artificial.

WEALTH VALUE OF LOWER VOLATILITY

Taking an economic lens to private asset smoothing, we can calculate the value of a smoothed private asset in terms of expected return. We begin with a mean-variance optimal investor.

$$\max\left(E[R] - \frac{\lambda}{2}Var[R]\right) \quad (1)$$

In Equation 1, the investor maximizes the expected return of their portfolio with a penalty for additional variance via a parameter $\boldsymbol{\lambda}$. As an illustrative example, we set $\boldsymbol{\lambda=2.5}$, the value where a 60/40 mix of stocks and bonds is the optimal portfolio (see appendix for details). With this value of $\boldsymbol{\lambda}$, an asset with 24% volatility (in line with the Russell 2000 Value Index) must have an expected return of 18% to have the same utility value as the Preqin buyout index, which has an expected return of 12% and a volatility of 10%. The value of the smoothed volatility of private equity is an extra 6 percentage points of return annually!

RARE DISASTER RISK

With few exceptions, the past two decades have seen a remarkable bull run in financial markets. Perhaps private assets contain a hidden jump risk that has yet to materialize, given the good market environment. The smoothed volatility observed within private assets' performance would be similar, then, to the performance of insurance-linked securities when no significant natural disasters have occurred. We could then quantify the hidden disaster risk embedded within private assets. Our approach is inspired by work such as Barro (2009), which showed how many asset pricing puzzles can be explained by the presence of rare unobserved jumps.[11]

10 We caution the reader from comparing the expected max drawdown to the one-touch probabilities. The one-touch probability can occur at any point within the three-year time horizon and is measured relative to the initial price. The calculation for the expected max drawdown defines a drawdown as the difference in log price or log return. Unlike a return, the log return is not bounded above -100% and can vary substantially from the return for extreme values.

11 Robert J. Barro, "Rare Disasters, Asset Prices, and Welfare Costs," *American Economic Review*, March 2009.

More formally, we assume that the return on a private asset has a probability of a discrete jump down.[12]

$$\frac{dS_t}{S_t} = \left(\mu + \frac{1}{2}\sigma^2\right) dt + \sigma dW_t + \left(e^{-\left(\mu+\frac{1}{2}\sigma^2+\delta\right)} - 1\right) dJ_t \quad (2)$$

In Equation 2, the return has two pieces: One is the standard geometric Brownian motion, and the other is a discontinuous Poisson jump process, $\boldsymbol{dJt}$, which we assume has intensity $\boldsymbol{p}$.[13] Over a one-year period, the asset has an expected log return of $\boldsymbol{\mu}$ conditional on the disaster not occurring and an expected log return of - $\boldsymbol{\delta}$ when exactly one jump occurs. The disaster has probability $\boldsymbol{p}$ of happening. We also return to our investor with mean-variance preferences.

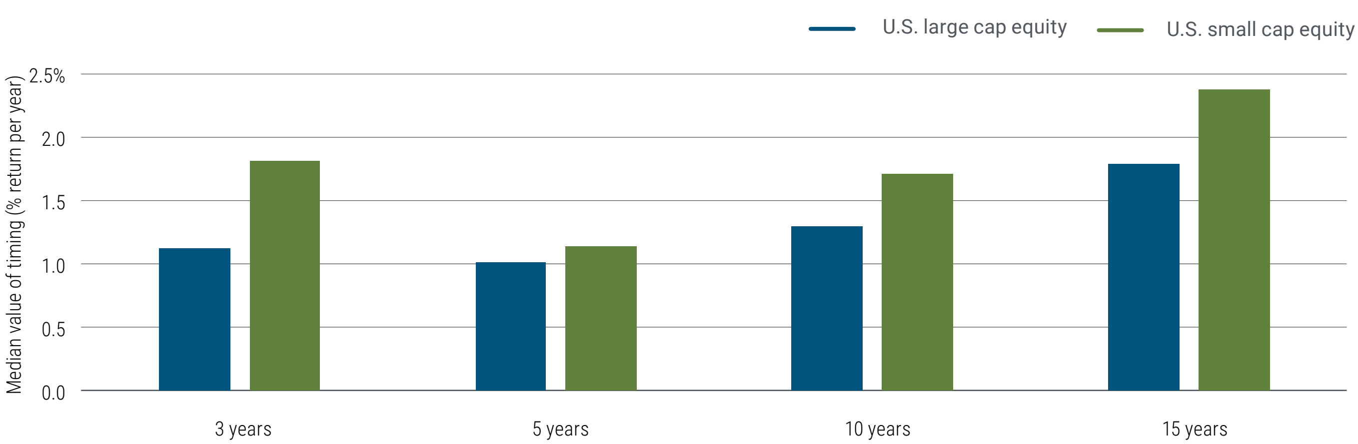
$$\max\left(E[R] - \frac{\lambda}{2} Var[R]\right) \quad (3)$$

To quantify the disaster risk, take our illustrative example of Preqin buyouts. We set $\boldsymbol{\mu=12\%}$ and $\boldsymbol{\sigma=10\%}$. We pick an illustrative value of $\boldsymbol{\delta=60\%}$, roughly in line with the maximum drawdown we observe for our custom benchmark. We can calculate the probability $\boldsymbol{p}$ such that the investor is indifferent about the choice between the private asset with hidden disaster and a similar asset with 12% expected return but 24% volatility, more in line with the Russell 2000 Value Index. We find $\boldsymbol{p=4.5\%}$. Thus, we find private assets need to hide a 4.5% chance of a 60% loss for the investor to be indifferent about the choice between the smoothed private asset and one with public-asset-like volatility. Given we have two decades of data, the probability that we haven't observed a jump down within our sample is $(1\text{-}.045)^{20}$, or approximately 40%. It is not impossible that there may be hidden jump risk within private assets.

VALUE OF REDUCING BEHAVIORAL BIASES

Since Tversky and Kahneman (1974)[14] first wrote that rational models were insufficient, people have been researching behavioral biases and how to prevent falling victim to them. One such bias is the panic selling of assets when markets are falling, even though expected returns are often highest precisely when prices are lowest. By locking up their money in private assets, a limited partner (LP) may reduce the loss in performance due to behavioral biases. Barber and Odean (2011)[15] document multiple suboptimal behaviors of investors, such as holding on to losers, limited attention and lack of diversification. This may seem contradictory to the concept of a liquidity premium, but this need not be the case.

Exhibit 10: Time-weighted versus money-weighted returns from Morningstar



Source: PIMCO and Morningstar as of December 2021. **For illustrative purposes only**. The exhibit is provided for illustrative purposes and is not indicative of the past or future performance of any PIMCO product.

12 See appendix for an explanation of the discrete-time equivalent process.
13 Within an interval $\boldsymbol{dt}$, the Poisson jump takes on values either 0 or 1 with probability 1-$\boldsymbol{pdt}$ and $\boldsymbol{pdt}$, respectively.
14 Amos Tversky and Daniel Kahneman, "Judgment under Uncertainty: Heuristics and Biases," *Science*, September 1974.
15 Brad M. Barber and Terrance Odean, "The Behavior of Individual Investors," working paper, September 2011.

In equilibrium, the liquidity premium should be equal to the forgone alpha an investor would have been able to generate if they had not locked up their capital. In other words, this premium will be a function of investor skill. For a sophisticated investor, Baz et al. (2021)[16] calculate a liquidity premium of around 2% per year. For the average investor, an inability to generate positive alpha is not unreasonable.

Using data from Morningstar, we attempt to quantify just how much the average investor may lose by trying to trade in and out of markets. In Exhibit 10, we show the median annualized difference between the total return and the Morningstar investor return for funds in the U.S. large cap equity and U.S. small cap equity categories. The Morningstar investor return is a dollar-weighted return that takes into account the cash inflows and outflows of the funds in each category. Interestingly, for all horizons the investor return is lower than the total return of the funds, meaning that the average investor generates negative returns by varying their exposure over time. For the 10-year horizon, roughly consistent with the life of buyout funds, we see there is a 1.7 percentage point per year tailwind from not adjusting the allocation to the U.S. small cap equity category. If we take small caps as representative of buyouts, we find investing in private equity may deliver 1.7 percentage points a year just by averting behavioral biases.

CONCLUSION

Although much less studied, the smoothed volatility profile of select private assets may offer benefits over public assets, which mark to market daily. Investors are inherently averse to risk, and the thought experiments we describe in this paper show just how much they may be willing to pay to avoid volatility. Even if private assets have the same expected return, and even if private assets' returns are not representative of the true risk underlying these investments, the smoothed performance that appears on quarterly statements can be powerful motivation to invest in private funds. With dry powder in private funds at record highs, better and more data likely becoming available with the passage of time, and public asset valuations trading in the tails, investors should carefully consider market conditions and suitability when evaluating private assets for their portfolios.

16 Jamil Baz, Steve Sapra, Christian Stracke and Wentao Zhao, "Valuing a Lost Opportunity: An Alternative Perspective on the Illiquidity Discount," *Journal of Portfolio Management,* February 2021.

17 The inverse of Shiller's CAPE, the option-adjusted spread on the Bloomberg Investment Grade Credit Index and the 10-year yield on U.S. TIPS are all in the bottom quartile relative to their history (1990-2021).

APPENDIX

Custom benchmark for private equity

Each month, we reevaluate the MSCI US universe of publicly traded equities. We first select firms that have an EV-to-EBITDA and an earnings yield that are above -100%, on average, over the past year to remove firms in bankruptcy. We then form a score for each stock by equally weighting the percentile rank within each industry based on market cap, net debt-to-EV, earnings yield and trailing one-year return volatility. We construct the custom index by taking a value-weighted average of stocks in the top quartile of scores within each industry at each month. The resulting custom index closely resembles publicly traded equities with the properties of companies bought out by private equity managers. Our custom benchmark has, on average, about 370 tickers, and the resulting volatility of the index matches the volatility of the public equity of firms that have debt trading in the high yield CDX index.

PROBABILITY OF GETTING FIRED

One-touch probability

Assume that the private asset follows a geometric Brownian motion with mean $\boldsymbol{\mu}$ and volatility $\boldsymbol{\sigma}$.

$$\frac{dS_t}{S_t} = \mu\, dt + \sigma\, dW_t \tag{A.1}$$

The probability of hitting a threshold within a certain period of time is referred to as a one-touch probability. For our geometric Brownian motion, this probability can be solved for in closed form.

$$Prob(inf_{0\leq t\leq T} S_t \leq \alpha) = \Phi\left(\frac{\ln\left(\frac{\alpha}{S_0}\right)-\left(\mu-\frac{\sigma^2}{2}\right)T}{\sigma\sqrt{T}}\right) + exp\left(\frac{2\ln\left(\frac{\alpha}{S_0}\right)\left(\mu-\frac{\sigma^2}{2}\right)}{\sigma^2}\right)\Phi\left(\frac{\ln\left(\frac{\alpha}{S_0}\right)+\left(\mu-\frac{\sigma^2}{2}\right)T}{\sigma\sqrt{T}}\right) \tag{A.2}$$

In Equation A.2, $\boldsymbol{\alpha}$ represents the minimum value threshold and $\boldsymbol{T}$ represents the time period. If we consider a special case where ***=σ^2/2***, the formula simplifies.

$$Prob(inf_{0\leq t\leq T} S_t \leq \alpha) = 2\Phi\left(\frac{\ln\left(\frac{\alpha}{S_0}\right)}{\sigma\sqrt{T}}\right) \tag{A.3}$$

Expected max drawdown

As in the previous section, assume that the private asset follows a geometric Brownian motion with mean $\boldsymbol{\mu}$ and volatility $\boldsymbol{\sigma}$.

$$\frac{dS_t}{S_t} = \mu\, dt + \sigma\, dW_t \tag{A.4}$$

The expected max drawdown for such a process was derived in Magdon-Ismail et al. (2004).

$$E(Max\ DD) = \begin{cases} \frac{\sigma^2}{\mu-\frac{1}{2}\sigma^2} Q_p\left(\frac{\left(\mu-\frac{1}{2}\sigma^2\right)^2}{\sigma^2}T\right) & if\ \mu > \frac{1}{2}\sigma^2 \\ 1.2552\sigma\sqrt{T} & if\ \mu = \frac{1}{2}\sigma^2 \\ -\frac{\sigma^2}{\mu-\frac{1}{2}\sigma^2} Q_n\left(\frac{\left(\mu-\frac{1}{2}\sigma^2\right)^2}{\sigma^2}T\right) & if\ \mu < \frac{1}{2}\sigma^2 \end{cases} \tag{A.5}$$

where $\boldsymbol{Q_p}$ and $\boldsymbol{Q_n}$ are integrals tabulated in the paper.

RETURN VALUE OF LOWER VOLATILITY

We use historical monthly data from 1999 to 2021 on the S&P 500 and the Bloomberg US Aggregate to generate the expected return and covariance matrix.

	S&P 500	US Aggregate
Return	7.94%	4.69%
Volatility	15.03%	3.39%
S&P correlation	1%	-0.04%

Source: PIMCO and Bloomberg. Volatility is measured as the standard deviation of monthly returns annualized over the sample period. Correlation is measured as the Pearson correlation coefficient using the monthly returns over the specified period.

Given these inputs, we know that we calibrate $\boldsymbol{\lambda}$ such that the optimal weights of a portfolio with no shorting and where the weights add up to 1 results in a 60/40 mix of stocks and bonds. In other words, we solve the following maximization:

$$\max_{\lambda} \mu_p - \frac{\lambda}{2}\sigma_p^2$$

$$\mu_p = (.6 \times .0794) + (.4 \times .0469)$$

$$.4] \begin{bmatrix} (.1503)^2 & (.1503)(.0339)(-.04) \\ (.1503)(.0339)(-.04) & (.0339)^2 \end{bmatrix} \begin{bmatrix} .6 \\ .4 \end{bmatrix} \tag{A.6}$$

The resulting value is ***λ=2.5***, which we use for our illustrative example.

RARE DISASTERS

In discrete time, the return process we use to model rare disasters is shown in Equation A.7.

$$R_{Private,t} = N(\mu, \sigma) - (\mu + \delta)\ Bern(p) \qquad \text{(A.7)}$$

We use ***Bern*** to denote the Bernoulli distribution with takes on values either 1 or 0 with probability $\boldsymbol{p}$ and $\boldsymbol{1\text{-}p}$, respectively. Over a one-year period, the asset has an expected return of $\boldsymbol{\mu}$ conditional on the disaster not occurring and an expected return of $\boldsymbol{-\delta}$ when the jump occurs.

Due to the dynamic nature of factors affecting markets, there is no guarantee that simulations will capture all relevant risk factors or that the implementation of any resulting solutions will protect against loss. All investments contain risk and may lose value. Simulated risk analysis contains inherent limitations and is generally prepared with the benefit of hindsight. Realized losses may be larger than predicted by a given model due to additional factors that cannot be accurately forecasted or incorporated into a model based on historical or assumed data.

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CMR2022-0223-2052399

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